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ORIGINAL PAPER

ASSESSING THE PROFITABILITY OF INVESTMENT PROJECTS USING ORDERED FUZZY NUMBERS

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ABSTRACT. **Background:** This article is motivated by the fact that most approaches to capital budgeting are deterministic. In reality, the capital budgeting problem is accompanied by the uncertainty and risk associated with dealing with imprecise data. Taking this uncertainty into account when performing analyses and calculations not only helps to better measure the profitability of investment projects, but also to expand the applicability of capital budgeting methods under real-life or uncertain conditions. The major contribution of this paper is the development of a novel approach to assessing the profitability of an investment project in the presence of uncertainty.

Methods: We present a novel approach for incorporating uncertainty into how the profitability of investment projects is assessed, which we term Ordered Fuzzy Net Present Value (OFNPV). The proposed method measures the level of investment project effectiveness using a model based on ordered fuzzy numbers (OFNs). In addition, ordered fuzzy numbers are used to describe changes to the investment parameters in the assumed time horizon. This paper illustrates an implementation of the proposed technique using a numerical example of an investment process in the logistics department of a company.

Results: The use of the proposed method based on OFNs allows experts to gauge the real-life accuracy of the considered phenomenon, and to express their assessment of its dynamic changes. This is vital to the problem of profitability assessment in investment projects.

Conclusions: Our approach offers a new perspective on the problem of investment in projects and constitutes an effective tool for assessing the profitability of investment projects. This tool could constitute a valuable source of knowledge for investors involved in decision-making processes.

Key words: project, investment project, capital budgeting, NPV, fuzzy number, ordered fuzzy number.

INTRODUCTION

We consider the following problem: there is a company and/or supply chain which has to decide whether or not to execute a potential investment in the area of logistics in a specified time horizon. It is assumed that the initial conditions needed to execute the investment project, including the initial costs, are known. The duration of execution of the investment is also known. What is not known, however, is the complete and unambiguous information about the market capitalization rate or the future inflows and outflows related to the execution of the logistics project in the considered time horizon. These quantities are determined by experts, including logisticians, based on their knowledge and experience. The analyzed problem comes down to finding out whether the given investment project is profitable for the entrepreneur under the specified conditions, taking into account the associated uncertainty and risk. Our aim is to numerically evaluate the considered investment project, which will allow the given company to decide whether to accept or reject it.

The above problem belongs to the class of complex problems, which involve imprecise data and whose solution requires the use of

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nonstandard mathematical methods. The need for a mathematical framework for describing imprecise phenomena that accompany the profitability assessment of investment projects (including estimating future money flows and market capitalization rate) was the reason for introducing the concept of ordered fuzzy numbers.

Due to their specific nature, investment projects undertaken in logistics are burdened with a degree of uncertainty and risk. The main reason for this is their long execution period (often several years). Deciding whether a given investment project should be carried out requires careful planning (either short- or long-range, depending on the planning horizon). It also requires foreseeing situations that could have a positive or negative impact on a particular investment. The issue of capital budgeting is inextricably linked with uncertainty and risk, which generally stem from the unavailability of certain data (i.e. dealing with data that is imprecise). Therefore, the conditions under which an investment project will be executed are hard to predict and define in a clear-cut manner. It must be emphasized that investment projects are already characterized by a high level of uncertainty at the outset. This is due to a key feature of projects, and long-term projects in particular, viz. their innovativeness. The level of knowledge associated with a project thus starts at virtually zero and increases as the project progresses. Only when all the effects, benefits, and costs are established, i.e. towards the end of the project, can the stipulated level of 100% certainty be reached.

Taking this uncertainty into account when performing analyses and calculations not only helps to better measure the profitability of investment projects, but also to expand the applicability of capital budgeting methods under real-life or uncertain conditions. The major contribution of this paper is the development of a novel approach to assessing the profitability of an investment project in the presence of uncertainty. This paper is organized as follows. The first section is devoted to a brief literature review. The third section outlines the employed research methodology, and Principal Component Analysis in particular. The subsequent section describes the resulting measures, discusses their validity and reliability, and presents our key findings. Section 5 contains conclusions and recommendations.

LITERATURE REVIEW

Assessing an investment project before it commences is a tedious task that requires the use of appropriate knowledge, modelling and forecasting methods, in addition to suitable mathematical techniques. The scope and the level of detail of such assessments can be varied, and they can be less or more formalized. Assessing project effectiveness is a crucial step in the decision-making process, during which investment projects are selected for execution. The results of such an assessment determine the subsequent stages of the investment, the analysis and allocation of resources, scheduling, budgeting and the control system. In practice, this involves the unpredictable behavior of the market in the project execution timeframe, including weather conditions, prices and costs, availability of resources, exchange rates, interest rates, behavior of the competition, changes in the level of supply and demand for a given product or service, etc. Traditionally, investment parameters (cash inflows and outflows, available investment capital) are presented in the form of crisp values. In the literature, one can find a variety of methods used for capital budgeting (see e.g. [Chansa-ngavej, Mount-Campbell 1991, Nosratpour et al. 2012]). The most commonly used methods include Payback Period (PP), Net Present Value (NPV), Profitability Index (PI), and the Internal Rate of Return (IRR). The classical forms of these methods do not take into account the uncertainty and risk which may be inherent in the information that they use as input. This information includes: future cash inflows, cash outflows and available investment capital, the required rate of return of an investment or the cost of capital, and the project duration [Kuchta 2000].

These shortcomings need to be addressed using novel and effective computing methods. Attempts to take into account both certain and uncertain data when considering the fuzzy environment of investments have been reported in the literature. We observe an increasing interest in the theory of fuzzy sets, which lays the foundations for describing uncertain events. Several authors already employ fuzzy set theory to help solve the capital budgeting problem in a fuzzy environment. The introduction of the concepts of fuzzy sets and fuzzy numbers was propelled by the need to mathematically describe imprecise and ambiguous phenomena. The above concepts were developed by Zadeh and described in [Zadeh 1965] as a generalization of the classical set theory. Motivated and inspired by this, several authors, such as Buckley [1987], Chiu, Park [1994, 1998], Chen [1991], Chansa-ngavej, Mount-Campbell [1991], Kuchta [2000], Li Calzi [1990], Huang [2007, 2008], Zhang et al. [2011], Tsao [2010], Kahraman and Kaya [2010], and Appadoo [2014] have investigated fuzzy set theory in the context of capital budgeting. In general, they used fuzzy numbers instead of crisp numbers in established formulas. Further works take uncertainty into consideration using fuzzy modelling to assess investment projects. Several authors reported the problems that arise when the capital budgeting problem is solved using fuzzy numbers [Buckley 1987, Calzi 1990; Kuchta 2000].

Recent papers have discussed the drawbacks of convex fuzzy numbers (CFNs) [Kosiński et al. 2009, 2013]. To overcome these issues, Kosiński developed the concept of ordered fuzzy numbers (OFNs). Interestingly, new interpretations offered by the ordered fuzzy numbers approach can be viewed as an extension of classical proposals. Methods for capital budgeting based on ordered fuzzy numbers remain relatively unexplored. Ordered fuzzy numbers were first used by Kosiński [2006] as a tool for a decision support system for evaluating financial projects. Kosiński tackled this problem using discount methods [Kosiński 2006; Chwastyk, Kosiński 2013, Kosiński et al. 2013]. The fuzzy equation determining the fuzzy internal rate of return has already been investigated for a class of OFNs with continuous branches in [Kosiński 2006]. Chwastyk and Kosiński [2013] used a class of Rational Ordered Fuzzy Numbers to model cash flow. The existence of a fuzzy solution for the IRR equation was proved based on the classical algebraic result that the root of a polynomial is a continuous function of the polynomial coefficients. Kosiński and his research team proposed a new tool for a decision support system for evaluating financial projects. It determined the IRR of an investment project in which all the expected expenditures and incomes are vague and are described by OFNs [Kosiński et al. 2013].

This new model of fuzzy numbers was capable of handling fuzzy inputs in the same way as real numbers, i.e. quantitatively. Crucially, the new interpretations provided by the OFN model constituted an extension of the classical proposals, and therefore it was not necessary to abandon established concepts to deal with new ones. In addition to the slightly different interpretation, this new model of fuzzy numbers exhibited many useful mathematical properties, in particular, it eliminated the main difficulty associated with classical fuzzy numbers, i.e. the unbounded loss of accuracy with each subsequent calculation. Moreover, this new approach made it possible to define new methods (based on the arithmetic of OFNs) for processing information in processes dealing with fuzzy control. During standard fuzzy arithmetic operations, certain problems arise during subtraction and division [Kosiński et al. 2013, Prokopowicz et al. 2017]. In recent publications it was demonstrated that the improved precision of operations and the possibility of solving equations in the set of ordered fuzzy numbers could help ameliorate these issues [Kosiński 2013, Chwastyk et al. 2015, Chwastyk, Kosiński 2013, Czerniak 2017, Roszkowska, Kacprzak 2016, Rudnik, Kacprzak 2016, Prokopowicz et al. 2017].

ORDERED FUZZY NUMBERS

A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which associates each point in $x \in X$ with a real number in the interval [0,1], i.e. the grade of membership of x in A. Thus, we can write $A = \{(x, \mu_A(x)); x \in X\}$,where $\mu_A : X \to [0,1]$ is the membership function of a fuzzy set. For each element $x \in X$ this function assigns its membership degree to fuzzy set A. A fuzzy number is a set that is defined in real numbers; it is convex, normal, it is described by a piecewise continuous membership function, and it has a bounded support [Kosiński et al. 2003]. A fuzzy number, and hence its membership function, can have two basic interpretations. It can be understood as a degree, to which x possesses a certain feature, or as a probability, with which a certain and, at this point, not entirely known value will assume a value x.

An ordered fuzzy number (OFN) A is an ordered pair of continuous functions (f, g), such that $f, g: [0,1] \to R$. The set of OFNs is denoted by $\mathcal{R}$. A small part of ordered fuzzy numbers corresponds to standard fuzzy numbers. A set of pairs of continuous functions, in which one function is increasing and the other is decreasing (while, simultaneously, the increasing function always assumes values lower or equal than the decreasing one) is a subset of a set of OFNs, which represents the class of all convex fuzzy numbers with continuous membership functions. They are termed proper OFNs. Figure 1 shows the construction of a membership function for such proper OFN (f, g).

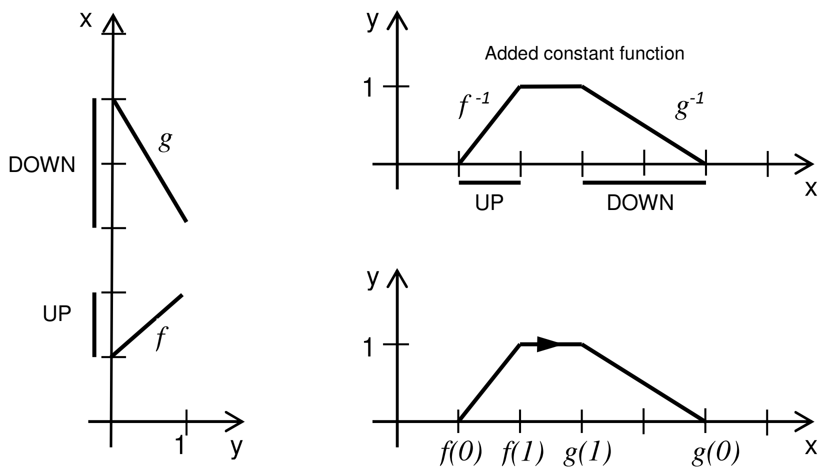

Fig. 1. Construction of a membership function for a proper OFN (f,g), where f is the increasing function

Figure 1 introduces the following notation: UP = $f([0,1])$ and DOWN = $g([0,1])$. Graphically, the curves of (f, g) and (g, f) are identical. However, these pairs of functions specify different ordered fuzzy numbers, and they differ in terms of their direction (denoted by an arrow in the diagrams).

OFNs share a significant feature, i.e. direction. OFNs allow us to describe an imprecise value in real-life processes [Kosiński et al. 2009]. The up- and down-branch parts of OFNs can be related to an expert's opinion concerning the dynamic changes of the analyzed value. It is possible to use OFNs without considering their direction, but taking it into account introduces additional information into operations performed with OFNs.

Let $A = (f_A, g_A)$, $B = (f_B, g_B)$, $C = (f_C, g_C)$ be ordered fuzzy numbers. The sum $C = A + B$, the product $C = A \times B$ and the division $C = A \div B$ are defined in the set $\mathcal{R}$ as follows:

$$f_C = f_A * f_B \text{ and } g_C = g_A * g_B \tag{1}$$

where "$*$" denotes "$+$", "$\times$", or "$\div$". Moreover, $A \div B$ is only defined when $f_B(y), g_B(y) \neq 0$ for all $y \in [0,1]$. In the OFN set, subtraction, exponentiation, and taking a root can also be defined in the usual manner, for instance:

$$(f, g)^n = (f^n, g^n) \tag{2}$$

The properties of ordered fuzzy numbers have been precisely described in the paper [Chwastyk, Kosiński 2013] and the monography [Prokowicz et al. 2017].

Each ordered fuzzy number A that is a pair of affine functions is uniquely determined by a 4-D vector composed of real numbers, while addition, subtraction and multiplication by a scalar are consistent with linear operations in the space of 4-D vectors (see Figure1):

$$[f(0), f(1), g(1), g(0)]. \qquad (3)$$

A proper linear ordered fuzzy number will be termed a trapezoidal ordered fuzzy number. If we further assume that $f(1) = g(1)$, we obtain a triangular ordered fuzzy number.

Defuzzification functionals, which map fuzzy numbers into real numbers, play a vital role in the application of ordered fuzzy numbers. The model of constructing a defuzzification functional presented in [Kosiński et al. 2009] allows us to obtain a number of defuzzification functionals, whether linear or non-linear. However, these functionals are not sensitive to direction, i.e. φ (f, g) = φ (g, f), and thus lack an essential feature of ordered fuzzy numbers. Defuzzification functionals sensitive to direction are considered in [Bednarek et al. 2014]. Here, on the other hand, a non-linear center of a gravity defuzzification functional was applied, which is defined by the following equations:

$$\phi_{COG}(\alpha, f, g) = \begin{cases} \dfrac{\int_0^1[(1-\alpha)f(s)+\alpha g(s)][f(s)-g(s)]ds}{2\int_0^1[f(s)-g(s)]ds}, & when \quad \int_0^1[f(s)-g(s)]ds \neq 0 \\ \dfrac{\int_0^1 f(s)ds}{\int_0^1 ds}, & when \quad \int_0^1[f(s)-g(s)]ds = 0 \end{cases} \qquad (4)$$

APPROACH TO INVESTMENT PROJECT DECISION-MAKING BASED ON ORDERED FUZZY NUMBERS

NPV is the most commonly used discount method. In essence, this method consists of an assessment of the present value of an investment project based on the forecasted streams of net cash flows, which are the measure of an investor's future benefits. NPV is defined as a sum of net cash flows (NCFs) discounted separately for each year and executed over the entire calculation period, with a constant level of interest (discount) rate. This value expresses the updated (on the day of assessment) value of benefits that an undertaking can yield in the future. The general form of NPV is:

$$NPV = \sum_{i=1}^{n} \frac{CF_i}{(1+k)^i} - N_0, \qquad (5)$$

where: n is the number of years, k is the market capitalization rate, CF_i is the cash flow in the i-th year of investment, and N_0 is the initial investment (outlay).

To define the generalization of NPV to ordered fuzzy numbers, let us assume that the initial outlay N_0 and the NPV are real numbers, whereas cash flows CF_i and the capitalization rate K are OFNs. The discounted cash flows in the i-th year of investment are calculated as follows:

$$DCF_i = \phi\left(\frac{CF_i}{((1,1)+K)^i}\right), \qquad (6)$$

where: $(1,1)$ are a pair of constant functions that assume the value of one, and $+, /$ signify, respectively, addition and division in the set of ordered fuzzy numbers defined by equation (2); exponentiation is performed according to equation (3), and ϕ is the defuzzification functional defined by equation (5). Then, equation (6) assumes the following form:

$$NPV = \sum_{i=1}^{n} DCF_i - N_0 . \qquad (7)$$

We propose an alternative to the conventional NPV method and present a new discount method, which we term Ordered Fuzzy Net Present Value (OFNPV). In this cash flow model, uncertain cash flows and the capitalization rate are specified as triangular ordered fuzzy numbers.

ILLUSTRATIVE EXAMPLE

Here, we consider an example of the execution of a potential investment project in the logistics department of a selected company. In order to define the conditions of investment project execution, an expert with appropriate knowledge and experience of planning and executing similar projects, e.g. a logistics manager, is involved in the decision-making process. The use of ordered fuzzy numbers poses a serious problem, namely that an expert is required to give an opinion on individual elements of the investment project in the form of ordered fuzzy numbers (pairs of functions).

It is assumed that the initial outlay N_0 will be 300,000 Arbitrary Monetary Units (AMU). The project is scheduled for 5 years. The remaining project parameters remain uncertain, and therefore, they are determined by an expert in the form of triangular ordered fuzzy numbers. The capitalization rate is $K = [0.11; 0.13; 0.13; 0.15]$. This means that according to the expert, capitalization rates lower than 11% or higher than 15% are not possible, whereas the value of 13% is the most probable one, and other values are probable to a different degree (the closer they are to 13%, the higher the probability). The difference between the proposed approach and the methods using classical fuzzy numbers consists in the incorporation of additional information through the OFN's direction. The direction of an OFN can be used to represent information about dynamic changes in the capitalization rate. Therefore, the ordered fuzzy capitalization rate K contains information about its rising tendency.

In a similar way, the expert determines the ordered fuzzy values of cash flows for subsequent years (cf. Table 1). The direction of OFNs is also used to represent complex information about cash flows. The ordered fuzzy cash flows in the first and third year of investment show a rising tendency, while the remaining ones – a decreasing tendency. This information is very important for the decision-making process.

For calculation purposes, the initial data have to be converted into pairs of functions so as to enable the use of arithmetic operations defined by formula (1). The triangular OFN $A = [a, b, b, c]$ corresponds to:

$$A_{OFN} = \big((b-a)x + a, (b-c)x + c\big), \qquad (8)$$

which is an ordered pair of linear functions.

Using the above formula, OFNs corresponding to values determined by the expert are defined. For instance, the capitalization rate expressed by ordered fuzzy numbers is: $K_{OFN} = (0.02x + 0.11; -0.02x + 0.15)$. The values of cash flows can be expressed analogously (cf. Table 1).

Table 1. Ordered fuzzy input data for the considered investment project

Investment year	Ordered fuzzy cash flows [a.m.u.]	Cash flow expressed as ordered fuzzy numbers
1	[75000,80000,80000,85000]	(5000x+75000, -5000x+85000)
2	[96000,93000,93000,90000]	(-3000x+96000, 3000x+90000)
3	[105000,111000,111000,118000]	(6000x+105000, -7000x+118000)
4	[126000,120000,120000,110000]	(-6000x+126000, 10000x+110000)
5	[130000,123000,123000,115000]	(-7000x+130000, 8000x+115000)

The calculation of discounted cash flows using OFNs will be presented for the first investment. The discounted cash flow in the first year will be:

$$\frac{CF_1}{(1,1)+K_{OFN}} = \frac{(5000x+75000, -5000x+85000)}{(1,1)+(0.02x+0.11; -0.02x+0.15)} =$$

$$= \frac{(5000x+75000, -5000x+85000)}{(0.02x+1.11; -0.02x+1.15)}$$

$$= \left(\frac{5000x+75000}{0.02x+1.11}, \frac{-5000x+85000}{-0.02x+1.15}\right),$$

where: CF_1 is the cash flow in the first year of investment and K_{OFN} is the capitalization rate. The next step in the proposed approach is to move from the discounted cash flow expressed by an ordered fuzzy number to a real value. This is achieved through the defuzzification functional according to equation (4): $\phi = \phi_{COG}\left(\frac{2}{3}, f, g\right)$.

$$DCF_1 = \phi\left(\frac{CF_1}{(1,1)+K_{OFN}}\right) = \phi\left(\frac{5000x+75000}{0.02x+1.11}, \frac{-5000x+85000}{-0.02x+1.15}\right) =$$

$$= \frac{\int_0^1 \left(\frac{1}{3}\cdot\frac{5000x+75000}{0.02x+1.11} + \frac{2}{3}\cdot\frac{-5000x+85000}{-0.02x+1.15}\right)\left(\frac{5000x+75000}{0.02x+1.11} - \frac{-5000x+85000}{-0.02x+1.15}\right)dx}{2\int_0^1 \left(\frac{5000x+75000}{0.02x+1.11} - \frac{-5000x+85000}{-0.02x+1.15}\right)dx}.$$

The discounted cash flows for the remaining periods of the project are determined analogously. The calculations shown here were made using MATLAB. Subsequently, these values undergo defuzzification using functional (4).The values are presented in Table 2 along with the NPV for the subsequent years of investment. In this case, NPV is equal to 56,019.1 AMU, which confirms the expected logistics project profitability.

Table 2. Investment discounted cash flows and NPV obtained using OFNs

Investment year	Discounted cash flows [a.m.u.]	NPV [a.m.u.]
0	300000.00	–300000.00
1	71473.40	–228526.60
2	71812.90	–156713.70
3	77202.70	–79511.00
4	70832.60	–8678.40
5	64697.50	56019.10
NPV		56019.10

The presented example demonstrates that the conclusions drawn from calculations employing OFNs are in agreement with current knowledge and economic analyses. Moreover, owing to the elimination of issues related to using classical fuzzy numbers, the model of ordered fuzzy numbers may prove to be a reliable tool for economic analysis and modelling.

ANALYSIS OF DYNAMICS OF CHANGES USING ORDERED FUZZY NUMBERS

This section considers an example of NPV inference based on the actual values of discounted net cash flows during the execution of a given investment project. In this case, the mathematical apparatus in the form of OFNs is employed to describe the changes in the values of discounted net cash flows and the dynamics of these changes. This approach is based on [Kosiński et al. 2009, Kacprzak 2014].

Investors are interested in how the expected NPV of an investment changes over a time period Δt with respect to the corresponding values for the scheduled period. Information of this kind can be gleaned through the use of triangular ordered fuzzy numbers, which enable the simultaneous presentation of planned and actual values of present cash flows. Triangular ordered fuzzy numbers can be represented by formula (3). We can use the direction of OFNs to represent complex information about the evolution of the planned (expected) discounted cash flows in relation to the actual ones.

The change in the discounted cash flow value in the i-th year of investment can be described by the following triangular OFN:

$$\overline{DCF_i} = [DCF_{0i}, \frac{DCF_{0i}+DCF_{ti}}{2}, \frac{DCF_{0i}+DCF_{ti}}{2}, DCF_{ti}], \quad (9)$$

where DCF_{0i} signifies the predicted discounted cash flow in the i-th year of investment, which is calculated using formula (6), whereas DCF_{ti} is the actual cash flow over the considered period. $\overline{DCF_i}$ allows us to see the graphical description of the change between the expected and real value of the discounted cash flow in the i-th year of investment. The width of the support reveals the magnitude of the change in discounted cash flows.

In addition, the goal is to facilitate an explanation of the change in the value of discounted cash flows by introducing an auxiliary quantity which characterizes this change, i.e. the change dynamics indicator (CDI). The CDI of discounted cash flows in the i-th year of investment can be represented by the following OFN:

$$dynDCF_i = \frac{1}{DCF_{0i}} \cdot \overline{DCF_i} = \left[\frac{DCF_{0i}}{DCF_{0i}}, \frac{DCF_{0i}+DCF_{ti}}{2DCF_{0i}}, \frac{DCF_{0i}+DCF_{ti}}{2DCF_{0i}}, \frac{DCF_{ti}}{DCF_{0i}}\right] =$$

$$\left[1, \frac{DCF_{0i}+DCF_{ti}}{2DCF_{0i}}, \frac{DCF_{0i}+DCF_{ti}}{2DCF_{0i}}, \frac{DCF_{ti}}{DCF_{0i}}\right]$$

Insight into the change dynamics of discounted cash flows is often crucial for evaluating the profitability of a given investment. By expressing the width of the support as a percentage

$$\left(\frac{DCF_{ti}}{DCF_{0i}} - 1\right) \cdot 100\%,$$

we can quantify the change in actual discounted cash flows in relation to the expected ones, with a positive value corresponding to an increase, and a negative value – to a decrease in the value of discounted cash flows. Figure 2 shows how the particularities of the change in the values of discounted cash flows over time can be interpreted through inspection. Growth trends correspond to positive direction, whereas downward trends – to negative direction. If the value of discounted cash flows remains unchanged, both graphs assume the same form, corresponding to a constant.

A graphical illustration of OFNs that mirrors the percentage change in discounted cash flows over time determined by the support width enables the rapid evaluation and ordering of individual periods during the execution of an investment project, e.g. from the change in cash flow that is the most unprofitable for the investor (the largest decrease in the discounted cash flow), to the most profitable one (corresponding to the highest increase). It now becomes possible to illustrate change dynamics in one chart, which facilitates evaluating how individual discounted cash flows change over time. However, a long planning horizon determined by the project execution period, together with the fact that discounted cash flows can either decrease, increase or remain unchanged over subsequent periods, hinder the evaluation of the resultant impact of the changes on the values of discounted cash flows and on their dynamics. OFNs facilitate information mining both for individual discounted cash flows and for the sum of discounted cash flows of a given investment.

The benefits of OFNs can be explained by applying them to the execution of an investment project example. Table 3 presents the expected and actual discounted cash flows expressed in AMU. NPV is assessed after three years of project execution in order to check the reliability of the results obtained during the previous phase (planning). According to Table 3, the discounted cash flow in the first year of project execution decreased from the expected level of 71,473.40 AMU to 65,300.00 AMU, i.e. by 8.64%. During the second and third year, however, an increase in the discounted cash flows was observed in relation to the expected flows, by 12.10% (8,688.10 AMU) and 12.30% (9,497.30 AMU), respectively (cf. Fig. 2).

The present discounted cash flows $\overline{DCF_1}$, $\overline{DCF_2}$, $\overline{DCF_3}$ expressed as OFNs and their change dynamics indices (also OFNs) are shown in Table 4.

Table 3. Expected and actual discounted cash flows for the considered investment project

Investment year	Expected discounted cash flow [a.m.u.]	Actual discounted cash flow [a.m.u.]
1	71473.40	65300.00
2	71812.90	80501.00
3	77202.70	86700.00
4	70832.60	no data
5	64697.50	no data

Table 4. Present discounted cash flows and their change dynamics indices for subsequent investment year

Investment year	Change of value of discounted cash flow [a.m.u.]	Change dynamics index
1	[71473.40;68386.70;68386.70;65300.00]	[1;0.957;0.957;0.914]
2	[71812.90;76159.95; 76156.95;80501.00]	[1; 1.06;1.06;1.121]
3	[77202.70;81951.35;81951.35;86700.00]	[1; 1.062;1.062;1.123]

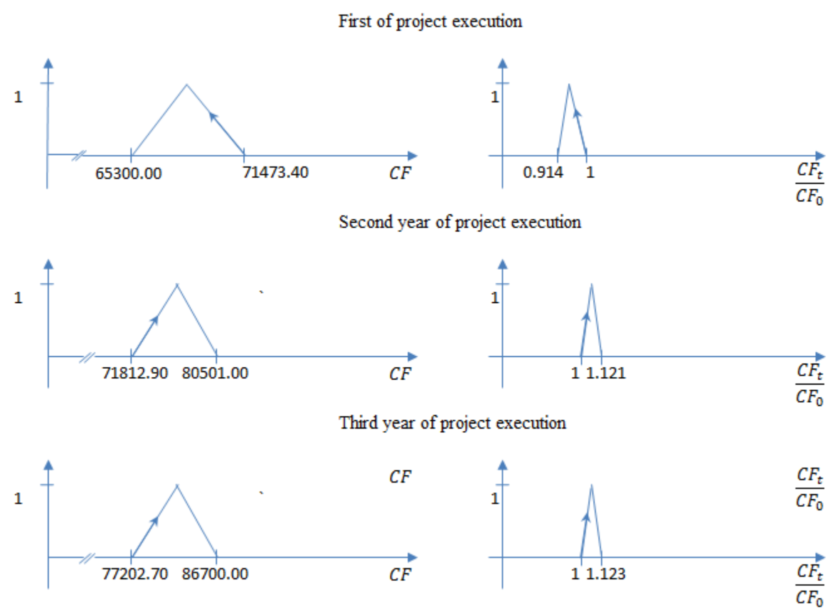

Fig. 2. Present discounted cash flows and their change dynamics for the considered investment

The change dynamics of discounted cash flows over the course of three years is expressed by means of an OFN, as the arithmetic mean of change dynamics over three subsequent years of project execution, yielding [1;1.026;1.026;1.053].

The next step in estimating the present cash flow in the fourth year of investment is to multiply the mean change dynamics by the scalar baseline for the fourth year of investment (70,832.60AMU), which yields [70,832.60;72,674.25;72,674.25;74,586.73]. Using forward inference from the data obtained over the first three years, the present cash flow for the fourth year of investment can be estimated at 74,586.73 AMU, and 68,126.47 AMU for the fifth year.

Examination of the data on the progress of the given investment indicates that the value of discounted cash flows changes. Deviations from expected values are taken into account when estimating the NPV of the investment. For this investment project, the current NPV, determined from the data recorded during project execution and from earlier forecasts, amounts to 75,214.20AMU. Therefore, in this case the increase in the predicted NPV is 34.27% (cf. Table 5). An increase in cash flows observed in the second and third year significantly influences the predicted NPV. The recorded values of cash flows reassure the investor that a correct decision has been made. The recorded values of cash flows reassure the investor that the correct decision has been made. The values predicted for the fourth and fifth years, based on the actual data, allow an updated value of NPV for the project to be inferred.

Table 5. Expected and actual cash flows and NPV for the investment

Investment year	Expected cash flows [a.m.u.]	Expected NPV [a.m.u.]	Present cash flows [a.m.u.]	Current NPV [a.m.u.]
0	300000.00	−300000.00	300000.00	−300000.00
1	71473.40	−228526.60	65300.00	−234700.00
2	71812.90	−156713.70	80501.00	−154199.00
3	77202.70	−79511.00	86700.00	−67499.00
4	70832.60	−8678.40	74586.73	7087.73
5	64697.50	56019.10	68126.47	75214.20
NPV		56019.10	–	75214.20

Actual cash flow values constitute valuable information for the investor. In the presented example, with the exception of the first year of investment, the investor witnesses a significant increase in cash flows. In economic practice, however, there exist investments, whose expected cash flows significantly deviate in minus from the actual values. In such cases it becomes necessary to diligently analyze the underlying causes of decreasing cash flows, and to make correct decisions – such as (as a last resort) halting the execution of the project in order to minimize losses incurred by the investor.

CONCLUSIONS

This article addresses the issue of assessing investment project profitability using OFNs. The main reasons for the complexity of this issue are: operating under conditions of uncertainty and the multi-criteria and multi-level nature of the decisions involved. To handle the uncertainty of net cash flows that stems from a lack of knowledge, this article proposes the use of ordered fuzzy numbers. They present imprecise data by means of a subjective possibility measurement associated with judgmental uncertainty, leading to a new approach for assessing the profitability of investment projects in fuzzy environments. The presented approach sheds new light on this common economic problem. The results of the assessment can be used by decision-makers to decide whether or not a given investment project ought to be carried out or rejected. By taking into account several options, the results of such an assessment can also facilitate selecting the most effective project, one that is deemed the most promising or otherwise favorable. When the investment cost is known, but the expected inflows remain hypothetical, the ability to make rational decisions is crucial.

Ordered fuzzy numbers may be used to illustrate information about cash flows and capitalization rates. They offer a clear, simultaneous representation of several pieces of information, while well-defined arithmetic operations on OFNs allow them to be aggregated. This article presents how to use OFNs to describe the change dynamics for given parameters in the assumed time horizon. Well-defined arithmetic operations and the direction properties of OFNs permit a modelling of the uncertainty associated with financial data and the construction of an entire decision support system in the future. The use of OFNs could eliminate several drawbacks of classical convex fuzzy numbers (CFNs), such as the loss of precision incurred by subsequent operations and the fact that even linear equations cannot be solved in a set of CFNs. In addition, a significant property of OFNs, their direction, is key to solving such problems and thus is valuable for decision-makers. By using OFNs, not only are experts able to assess the degree to which they recognize the considered phenomenon to be accurate and true to life, but also to express their assessment of its dynamics. This is key to assessing the profitability of investment projects. Every investor is interested in how the values of present cash flows and capitalization rates may change compared to their corresponding baseline values (t=0). The proposed approach based on OFNs simultaneously presents the values of present cash flows and capitalization rates over the period under study, and at the baseline.

An illustrative example was presented to emphasize the advantages of the proposed approach. This is an efficient, qualitative-quantitative approach to measurement and evaluation of investment project effectiveness based on OFNs. The proposed approach is easy to interpret by decisions-makers. It offers a considerable advantage when strategic decisions concerning investment projects are made. The authors intend to continue the search for a computationally efficient, qualitative-quantitative approach to capital budgeting under the conditions of uncertainty. Moreover, we intend to develop the approach presented in our previous article by transferring defuzzification to a different stage of calculations. Further discount methods (profitability index and internal rate of return) will be presented for a more precise evaluation of alternative investment projects. Such tools can be perceived as a decision support system based on OFNs. We plan to provide OFN-based commercial software dedicated to the investment process, which we envision will find uses in both practical and academic applications alike.

The presented approach to a profitability analysis of investment projects can be viewed as an early warning system, whose aim is to analyze signals from the environment and to interpret them correctly. It can serve as a tool for detecting potential opportunities and risks in the life cycle of investment projects. This tool could constitute a valuable source of knowledge for investors involved in decision-making processes. The evidence in favor of an ordered fuzzy approach highlights the advantages over the original crisp version of capital budgeting methods. The presented approach provides a formulation that more closely conforms to real situations. The proposed OFNPV method does not imply rejection of other discounted cash formulations but rather compliments existing fuzzy model formulations.

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OCENA OPŁACALNOŚCI PROJEKTÓW Z WYKORZYSTANIEM SKIEROWANYCH LICZB ROZMYTYCH

STRESZCZENIE. **Wstęp:** U podstaw rozważań leży stwierdzenie, że większość podejść do budżetowania kapitałowego ma charakter deterministyczny. W rzeczywistości problemowi budżetowania towarzyszy niepewność i ryzyko związane z przetwarzaniem nieprecyzyjnych danych. Uwzględnienie tej niepewności nie tylko pomaga lepiej zmierzyć efektywność projektów inwestycyjnych, ale także rozszerzyć zastosowanie metod budżetowania kapitałowego w warunkach rzeczywistych lub niepewnych. Głównym celem artykułu jest opracowanie nowatorskiego podejścia do oceny opłacalności projektu inwestycyjnego w warunkach niepewności.

Metody: Prezentujemy nowatorskie podejście uwzględniające niepewność w ocenie opłacalności projektów inwestycyjnych Ordered Fuzzy Net Present Value (OFNPV). Proponowana metoda umożliwia dokonanie oceny efektywności projektu inwestycyjnego za pomocą modelu opartego na skierowanych liczbach rozmytych (OFN). Ponadto skierowane liczby rozmyte służą do opisu zmian parametrów inwestycyjnych w założonym horyzoncie czasowym. Artykuł ilustruje wdrożenie proponowanego podejścia z wykorzystaniem przykładu procesu inwestycyjnego w przedsiębiorstwie w obszarze logistycznym.

Wyniki: Zastosowanie proponowanej metody opartej na OFN pozwala ekspertom ocenić dokładność rozpatrywanego zjawiska, a także wyrazić swoją ocenę na temat dynamiki ich zmian. Ma to kluczowe znaczenie dla problemu oceny opłacalności projektów inwestycyjnych.

Wnioski: Proponowane podejście oferuje nowe spojrzenie na problem inwestycyjny i stanowi skuteczne narzędzie do oceny opłacalności projektów inwestycyjnych. Narzędzie to może stanowić cenne źródło wiedzy dla inwestorów zaangażowanych w procesy decyzyjne.

Słowa kluczowe: projekt, projekt inwestycyjny, budżetowanie kapitałowe, NPV; liczba rozmyta, skierowana liczba rozmyta.

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