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ORIGINAL PAPER

# TWO-STORAGE INVENTORY MODEL FOR DETERIORATING ITEMS WITH PRICE DEPENDENT DEMAND AND SHORTAGES UNDER PARTIAL BACKLOGGED IN FUZZY APPROACH

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**ABSTRACT**. **Background:** In this paper we developed a fuzzy two-warehouse (one is OW, the own warehouse and other is RW, the rented warehouse) inventory model of deteriorating items with price dependent demand rate and allowed shortages under partially backlogged conditions. Since the capacity of any warehouse is limited, the supplier has to rent a warehouse for keeping the excess units over the fixed capacity W of the own warehouse in practice. The rented warehouse owed higher holding cost than the own warehouse. In this paper we considered holding cost, deterioration rate, shortages cost and lost sales as triangular fuzzy numbers.

**Methods:** Graded Mean Integration Representation is used to defuzzify the total cost function. The result obtained by this method is compared with crisp model with the help of a numerical example. Sensitivity analysis is accomplished to changing one parameter at a time and keeping others at their archetypal.

**Results and conclusions**: It has been proved that graded mean integration representation method gives more accurate result as compare to crisp model.

**Key words:** Inventory, Two-Warehouse System, Deterioration, Shortages, Triangular Fuzzy Number, Graded Mean Integration Representation Method.

## INTRODUCTION

Now-a-days the modern business environment is highly competitive in nature due to the globalization of market. The suppliers need large number of customer day by day for the increase of their business. To attract a large number of customers, an organization grabs a huge collection of goods which cannot be stored indefinitely in its own warehouse with limited capacity. Hence the organization hires one or more warehouse in rental basis to store the excess units of goods. Usually, the rented warehouse charges higher unit holding cost than the own warehouse.

In the last few decades, two-warehouse inventory models have been widely applied in business world. In 1976, Hartely first presented a basic two-warehouse model, where the cost of transporting a unit from rented warehouse to owned warehouse was not considered. Then, Sarma [1983] extended Hartely's model by introducing the transportation cost. After his pioneering contribution, several other researchers have attempted to extend their works to various other realistic situations. Goswami and Chaudhuri [1992] developed an economic order quantity model for items with two levels of storage for linear demand. Hence researchers attracted towards the development of two-warehouse inventory model considering



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different parameters. Roy et al. [2018] developed a probabilistic inventory model of deteriorating items for adopting the trade-credit policy for their customers those who are interested for their backorder demands. Chakrabarty et al. [2018] investigated a two warehouse inventory model for single deteriorating items under assumption of shortages, and partial backlogging under payments delay where the effect of inflation and time value for money is also incorporated. Mandal and Giri [2018] explored a single buyer, two warehouse inventory model with imperfect production process of the vendor. The demand depends upon the stock where the vendor offers a quantity discount to motivate the customers to buy more quantities. A credit policy approach in a two warehouse inventory model for deteriorating items with price and stock-dependent demand under partial backlogging is explained by Panda et al.[2019]. For more research in this field see Benkherouf [1997], Yang [2004], Huang [2006], Lee and Hsu [2009], Liang and Zhou [2011], Yang and Chang [2013], Jaggi et al. [2013, 2017], Bhunia et al. [2014, 2015], Xu et al. [2016], Sheikh and Patel [2017] etc.

A large number of research papers have been studied in the area of two-warehouse inventory model in crisp approach. However, a very few researchers extended their works to fuzzy environment. In this above literature review, we discussed most of the fuzzy two-warehouse inventory models. Customer satisfaction plays a crucial role for an organization in the present competitive market scenario to maximizing the profit. In this context, the inventory level should be properly set as to meet the customer's expectations. With a lost sale, the customer's needs for the item is filled by a competitor which is assumed as the lost of profit in sales. On the other hand, the organization not only loses the customer but also loses the customer's goodwill. Therefore we did not exclude the stock out cost from the total profit. No organization ignores the effect of demand in its business. There are many types of inventories as per time, price, stock etc. At the end of each calendar year for a product, the demand is same among the customer. The scarcity of a product in the market increases its demand. During this period the demand is depends upon the selling price. As we increase the selling price, the demand is decreases and vice-versa. By considering the above parameters into account, we developed a two-warehouse inventory model for deteriorating items with price dependent demand where shortages are partially backlogged. Here, we have introduced two warehouses- one own warehouse and other is rented warehouse. The holding cost of rented warehouse charges are higher than that of own warehouse. The cost components (holding cost, shortage cost, lost sales) and deterioration rates for two warehouses are assumed as triangular fuzzy numbers. The objective of this model is to maximize the profit and minimize the total cost. Graded Mean Integration Representation Method is used for defuzzification of the total cost function. The inventory total cost is obtained in both crisp and fuzzy environment. Numerical example is given to illustrate the validity of the given model. Sensitivity analysis is also carried out by the help of Mathematica 11.1 software to analyze the effect of changes of each parameter by keeping other parameters at their original.

The remainder of this paper is organized as follows. In sect. 2, assumptions of the proposed model are given. Notations of the model are provided in sect. 3. Mathematical model in Crisp and Fuzzy sense is formulated in sect. 4. In sect. 5, numerical example is illustrated to support the proposed model. Sensitivity analysis is carried out by using Mathematica software in sect. 6 followed by conclusion in sect. 7.

## ASSUMPTIONS

1. The inventory system involves only one item.
2. The replenishment occurs instantaneously at infinite rate.
3. The lead time is negligible.
4. The demand rate is a function of selling price.
5. The shortages are allowed and partially backlogged.
6. The own warehouse has a limited capacity of W units.

7. The rented warehouse has unlimited capacity calculated per day basis.
8. The holding unit cost of RW is greater than that of OW.
9. The items assumed in this model are deteriorating in nature.
10. Neglecting the higher power of θ.
11. The items are stored in OW first.
12. The items kept in RW will be consumed first.

## NOTATIONS

| | |
|---|---|
| $I_r(t)$ | Inventory level at time $t$ in RW, $t \geq 0$ |
| $I_o(t)$ | Inventory level at time $t$ in OW, $t \geq 0$ |
| $\theta$ | Rate of deterioration |
| $\alpha$ | Initial demand rate |
| $\beta$ | Positive demand parameter |
| $t_1$ | Time point when stock level of RW reaches to zero |
| $t_2$ | Time point when stock level of OW reaches to zero |
| $W$ | Storage capacity of OW |
| $C_1$ | Selling price per unit |
| $S$ | Initial stock level |
| $q_1$ | Backorder quantity during stock out |
| $T$ | Cycle time |
| $p$ | Purchasing cost ($/unit/day). |
| $k$ | Rate of backlogging |
| $h_r$ | Holding cost ($/unit/year) in RW |
| $h_o$ | Holding cost ($/unit/year) in OW |
| $d$ | Unit deterioration cost ($/unit/day) |
| $C_2$ | Unit shortage cost ($/unit/day). |
| $C_3$ | Unit lost sale cost ($/unit/day). |
| $TC(t_1, t$ | Total average cost ($/unit/day). |
| $\tilde{\theta}$ | Fuzzy deterioration rate |
| $\tilde{h}_r$ | Fuzzy holding cost ($/unit/day) in RW |
| $\tilde{h}_o$ | Fuzzy holding cost ($/unit/day) in OW |
| $\tilde{C}_2$ | Fuzzy shortage cost ($/unit/day) |
| $\tilde{C}_3$ | Fuzzy opportunity cost due to lost sale ($/unit/day) |
| $\widetilde{TC}(t_1, t$ | Fuzzy total cost ($/unit/day) |
| $\widetilde{TC}_G(t_1,$ | Defuzzified value of $\widetilde{TC}(t_1, t_2)$ by applying GMIR method |

## MATHEMATICAL FORMULATIONS

Suppose $q$ units are received in the stock at the beginning from which $q_1$ units are utilized to satisfy backlogged demand and $S$ units are the initial stock level. First of all $W$ units of material stored in OW and then rest $(S - W)$ units are stored in a RW. Since holding cost $h_r$ of RW is greater than the holding cost $h_o$ of OW, the items in RW will be consumed first. During the consumption period of RW, the inventory level of OW is decreased due to deterioration only. At time $t = t_1$, the inventory level of RW becomes zero due to demand and deterioration. During $[t_1, t_2]$ stock is available only in OW. At time $[t = t_2]$; inventory level of OW depletes to zero due to demand and deterioration and after that shortage occurs. This is shown in the Fig. 1.

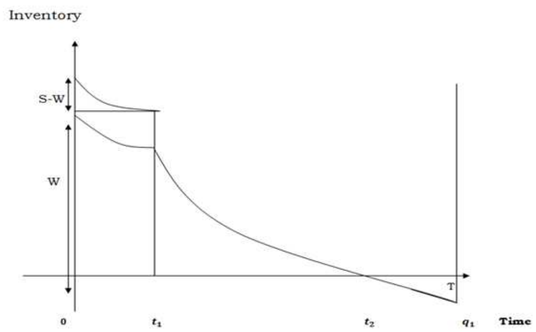


Fig. 1. Inventory time graph of two-storage inventory model

## CRISP MODEL

The differential equations for this inventory level is describing as follows.

The differential equations for this inventory level is describing as follows:

$$\frac{dI_r(t)}{dt} = -\theta I_r(t) - (\alpha - \beta C_1), \quad 0 \le t \le t_1 \tag{1}$$

with $I_r(t_1) = 0$

$$\frac{dI_o(t)}{dt} = -\theta I_o(t), \qquad 0 \le t \le t_1 \tag{2}$$

with $I_o(0) = W$

$$\frac{dI_o(t)}{dt} = -\theta I_o(t) - (\alpha - \beta C_1), \quad t_1 \le t \le t_2 \tag{3}$$

with $I_o(t_2) = 0$

The solutions of the above equations (1), (2) and (3) are given by

$$I_r(t) = \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta(t_1-t)} - 1\right), \quad 0 \le t \le t_1 \tag{4}$$

$$I_o(t) = We^{-\theta t}, \qquad 0 \le t \le t_1 \tag{5}$$

$$I_o(t) = \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta(t_2-t)} - 1\right), \quad t_1 \le t \le t_2 \tag{6}$$

From (5), we have

$$I_r(0) = S - W$$

$$S = W + \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_1} - 1\right) \tag{7}$$

At $t = t_1$, Equation (5) and (6) yields

$$We^{-\theta t_1} = \frac{(\alpha - \beta C_1)}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)$$

$$W = \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_2} - e^{\theta t_1}\right) \tag{8}$$

Purchasing cost (P.C)

$$P.C = (S + q_1)p$$

$$q_1 = \int_{t_2}^{T} k(\alpha - \beta C_1)dt = (\alpha - \beta C_1)k(T - t_2)$$

$$P.C = \left\{\left(W + \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_1} - 1\right)\right) + (\alpha - \beta C_1)k(T - t_2)\right\}p \tag{9}$$

Holding cost (H.C)

$$H.C = H.C_r + H.C_o$$

$$H.C_r = h_r \int_0^{t_1} I_r(t)dt = h_r \frac{(\alpha-\beta C_1)}{\theta}\left(\frac{e^{\theta t_1 - 1}}{\theta} - t_1\right) \tag{10}$$

$$H.C_o = h_o\left\{\int_0^{t_1} I_o(t)dt + \int_{t_1}^{t_2} I_o(t)dt\right\}$$

$$= \frac{Wh_o}{\theta}\left(1 - e^{-\theta t_1}\right) + h_o\frac{(\alpha-\beta C_1)}{\theta}\left(\frac{e^{\theta(t_2-t_1)-1}}{\theta} + (t_1 - t_2)\right) \tag{11}$$

Deterioration cost (D.C)

$$D.C = D.C_r + D.C_o$$

$$D.C_r = d\left\{\frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_1} - 1\right) - (\alpha - \beta C_1)t_1\right\} \tag{12}$$

$$D.C_o = d\{W - (\alpha - \beta C_1)(t_2 - t_1)\} \tag{13}$$

Shortage cost (S.C)

$$S.C = C_2\int_{t_2}^{T}(\alpha - \beta C_1)dt = C_2(\alpha - \beta C_1)(T - t_2) \tag{14}$$

Lost sale cost (L.C)

$$L.C = C_3\int_{t_2}^{T}(1 - k)(\alpha - \beta C_1)dt = C_3(1 - k)(\alpha - \beta C_1)(T - t_2) \tag{15}$$

Total average cost $TC(t_1, t_2)$ for this model during a cycle is given by

$$TC(t_1, t_2) = \frac{1}{T}[P.C + H.C + D.C + S.C + L.C]$$

$$
\begin{aligned}
= \frac{1}{T}\Bigg[ & \left\{\left(W + \frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_1}-1\right)\right) + (\alpha-\beta C_1)k(T-t_2)\right\}p + h_r\frac{(\alpha-\beta C_1)}{\theta}\left(\frac{e^{\theta t_1}-1}{\theta}-t_1\right) \\
& + \frac{W h_o}{\theta}\left(1-e^{-\theta t_1}\right) + h_o\frac{(\alpha-\beta C_1)}{\theta}\left(\frac{e^{\theta(t_2-t_1)}-1}{\theta} + (t_1-t_2)\right) \\
& + d\left\{\frac{(\alpha-\beta C_1)}{\theta}\left(e^{\theta t_1}-1\right) - (\alpha-\beta C_1)t_1 + W - (\alpha-\beta C_1)(t_2-t_1)\right\} \\
& + C_2(\alpha-\beta C_1)(T-t_2) + C_3(1-k)(\alpha-\beta C_1)(T-t_2)\Bigg]
\end{aligned}
\tag{16}
$$

To minimize the total cost function $TC(t_1, t_2)$ per unit time, the optimum value of $t_1$ and $t_2$ can be obtained by solving the following equations:

$$
\frac{\partial TC(t_1,t_2)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC(t_1,t_2)}{\partial t_2} = 0 \tag{17}
$$

Equation (17) is equivalent to

$$
\left[\frac{\left\{\begin{array}{c} d e^{\theta t_1}(\alpha-\beta C_1) + e^{\theta t_1}p(\alpha-\beta C_1) + W h_o e^{-\theta t_1} \\ + h_o\left(1-e^{\theta(t_2-t_1)}\right)(\alpha-\beta C_1) + \dfrac{h_r\left(e^{\theta t_1}-1\right)(\alpha-\beta C_1)}{\theta^2} \end{array}\right\}}{T}\right] = 0
$$

and

$$
\left[\frac{\left\{\begin{array}{c} -kp(\alpha-\beta C_1) + d(-\alpha+\beta C_1) - C_2(\alpha-\beta C_1) \\ -C_3(1-k)(\alpha-\beta C_1) + \dfrac{h_o\left(e^{\theta(t_2-t_1)}-1\right)(\alpha-\beta C_1)}{\theta} \end{array}\right\}}{T}\right] = 0
$$

Provided it satisfies the equations

$$
\begin{aligned}
& \frac{\partial^2 TC(t_1,t_2)}{\partial t_1^2} > 0\,, \quad \frac{\partial^2 TC(t_1,t_2)}{\partial t_2^2} > 0 \quad \text{and} \\
& \left(\frac{\partial^2 TC(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,t_2)}{\partial t_2^2}\right) - \left(\frac{\partial^2 TC(t_1,t_2)}{\partial t_1^2 \partial t_2^2}\right)^2 > 0
\end{aligned}
\tag{18}
$$

## FUZZY MODEL

Due to uncertainty in nature it is not easy to define all the system parameters exactly, subsequently let us assume that some of these parameters namely $\tilde{h}_r$ , $\tilde{h}_o$ , $\tilde{C}_2$ , $\tilde{C}_3$, $\tilde{\theta}$ may change within some limits.

Suppose $\tilde{h}_r = (r_1, r_2, r_3)$, $\tilde{h}_o = (O_1, O_2, O_3)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$, $\tilde{C}_2 = (n_1, n_2, n_3)$, and $\tilde{C}_3 = (l_1, l_2, l_3)$ be consider as triangular fuzzy numbers.

Then the total average cost is given by

$$\widetilde{TC}(t_1,t_2) = \frac{1}{T}\left[\begin{array}{c} \left\{\left(W + \frac{(\alpha-\beta C_1)}{\tilde{\theta}}\left(e^{\tilde{\theta} t_1} - 1\right)\right) + (\alpha - \beta C_1)k(T - t_2)\right\}p \\ +\tilde{h}_r \frac{(\alpha-\beta C_1)}{\tilde{\theta}}\left(\frac{e^{\tilde{\theta} t_1}-1}{\tilde{\theta}} - t_1\right) + \frac{W\tilde{h}_o}{\tilde{\theta}}\left(1 - e^{-\tilde{\theta} t_1}\right) \\ +\tilde{h}_o \frac{(\alpha-\beta C_1)}{\tilde{\theta}}\left(\frac{e^{\tilde{\theta}(t_2-t_1)}-1}{\tilde{\theta}} + (t_1 - t_2)\right) \\ +d\left\{\frac{(\alpha-\beta C_1)}{\tilde{\theta}}\left(e^{\tilde{\theta} t_1} - 1\right) - (\alpha - \beta C_1)t_1 + W - (\alpha - \beta C_1)(t_2 - t_1)\right\} \\ +\tilde{C}_2(\alpha - \beta C_1)(T - t_2) + \tilde{C}_3(1 - k)(\alpha - \beta C_1)(T - t_2) \end{array}\right] \quad (19)$$

We defuzzified the fuzzy total cost function $\widetilde{TC}(t_1,t_2)$ by Graded Mean Integration Representation (GMIR) method as follows:

$$\widetilde{TC}_G(t_1,t_2) = \frac{1}{6}\left[\widetilde{TC}_{G1}(t_1,t_2) + 4\,\widetilde{TC}_{G2}(t_1,t_2) + \widetilde{TC}_{G3}(t_1,t_2)\right] \quad (20)$$

where

$$\widetilde{TC}_{G1}(t_1,t_2) = \frac{1}{T}\left[\begin{array}{c} \left\{\left(W + \frac{(\alpha-\beta C_1)}{\theta_1}\left(e^{\theta_1 t_1} - 1\right)\right) + (\alpha - \beta C_1)k(T - t_2)\right\}p \\ +r_1 \frac{(\alpha-\beta C_1)}{\theta_1}\left(\frac{e^{\theta_1 t_1}-1}{\theta_1} - t_1\right) + \frac{WO_1}{\theta_1}\left(1 - e^{-\theta_1 t_1}\right) \\ +O_1 \frac{(\alpha-\beta C_1)}{\theta_1}\left(\frac{e^{\theta_1(t_2-t_1)}-1}{\theta_1} + (t_1 - t_2)\right) \\ +d\left\{\frac{(\alpha-\beta C_1)}{\theta_1}\left(e^{\theta_1 t_1} - 1\right) - (\alpha - \beta C_1)t_1 + W - (\alpha - \beta C_1)(t_2 - t_1)\right\} \\ +n_1(\alpha - \beta C_1)(T - t_2) + l_1(1 - k)(\alpha - \beta C_1)(T - t_2) \end{array}\right]$$

$$\widetilde{TC}_{G2}(t_1,t_2)=\frac{1}{T}\begin{bmatrix} \left\{\left(W+\frac{(\alpha-\beta C_1)}{\theta_2}\left(e^{\theta_2 t_1}-1\right)\right)+(\alpha-\beta C_1)k(T-t_2)\right\}p \\ +r_2\frac{(\alpha-\beta C_1)}{\theta_2}\left(\frac{e^{\theta_2 t_1}-1}{\theta_2}-t_1\right)+\frac{WO_2}{\theta_2}\left(1-e^{-\theta_2 t_1}\right) \\ +O_2\frac{(\alpha-\beta C_1)}{\theta_2}\left(\frac{e^{\theta_2(t_2-t_1)}-1}{\theta_2}+(t_1-t_2)\right) \\ +d\left\{\frac{(\alpha-\beta C_1)}{\theta_2}\left(e^{\theta_2 t_1}-1\right)-(\alpha-\beta C_1)t_1+W-(\alpha-\beta C_1)(t_2-t_1)\right\} \\ +n_2(\alpha-\beta C_1)(T-t_2)+l_2(1-k)(\alpha-\beta C_1)(T-t_2) \end{bmatrix}$$

$$\widetilde{TC}_{G3}(t_1,t_2)=\frac{1}{T}\begin{bmatrix} \left\{\left(W+\frac{(\alpha-\beta C_1)}{\theta_3}\left(e^{\theta_3 t_1}-1\right)\right)+(\alpha-\beta C_1)k(T-t_2)\right\}p \\ +r_3\frac{(\alpha-\beta C_1)}{\theta_3}\left(\frac{e^{\theta_3 t_1}-1}{\theta_3}-t_1\right)+\frac{WO_3}{\theta_3}\left(1-e^{-\theta_3 t_1}\right) \\ +O_3\frac{(\alpha-\beta C_1)}{\theta_3}\left(\frac{e^{\theta_3(t_2-t_1)}-1}{\theta_3}+(t_1-t_2)\right) \\ +d\left\{\frac{(\alpha-\beta C_1)}{\theta_3}\left(e^{\theta_3 t_1}-1\right)-(\alpha-\beta C_1)t_1+W-(\alpha-\beta C_1)(t_2-t_1)\right\} \\ +n_3(\alpha-\beta C_1)(T-t_2)+l_3(1-k)(\alpha-\beta C_1)(T-t_2) \end{bmatrix}$$

To minimize the total cost function $\widetilde{TC}_G(t_1,t_2)$ per unit time, the optimum value of $t_1$ and $t_2$ can be obtained by solving the following equations:

$$\frac{\partial\widetilde{TC}_G(t_1,t_2)}{t_1}=0 \quad \text{and} \quad \frac{\partial\widetilde{TC}_G(t_1,t_2)}{t_2}=0 \tag{21}$$

Equation (21) is equivalent to

$$\begin{bmatrix} \left\{\frac{de^{\theta_1 t_1}(\alpha-\beta C_1)+p(\alpha-\beta C_1)e^{\theta_1 t_1}+WO_1e^{-\theta_1 t_1}+\frac{\left(e^{\theta_1 t_1}-1\right)(\alpha-\beta C_1)r_1}{\theta_1^2}+\frac{O_1(\alpha-\beta C_1)\left(1-e^{\theta_1(t_2-t_1)}\right)}{\theta_1}}{6T}\right\} \\ +4\left\{\frac{de^{\theta_2 t_1}(\alpha-\beta C_1)+p(\alpha-\beta C_1)e^{\theta_2 t_1}+WO_2e^{-\theta_2 t_1}+\frac{\left(e^{\theta_2 t_1}-1\right)(\alpha-\beta C_1)r_2}{\theta_2^2}+\frac{O_2(\alpha-\beta C_1)\left(1-e^{\theta_2(t_2-t_1)}\right)}{\theta_2}}{T}\right\} \\ +\left\{\frac{de^{\theta_3 t_1}(\alpha-\beta C_1)+p(\alpha-\beta C_1)e^{\theta_3 t_1}+WO_3e^{-\theta_3 t_1}+\frac{\left(e^{\theta_3 t_1}-1\right)(\alpha-\beta C_1)r_3}{\theta_3^2}+\frac{O_3(\alpha-\beta C_1)\left(1-e^{\theta_3(t_2-t_1)}\right)}{\theta_3}}{T}\right\} \end{bmatrix}=0$$

and

$$\left[\begin{array}{l}\left\{\dfrac{\begin{array}{c}-kp(\alpha-\beta C_1)+d(\beta C_1-\alpha)-(1-k)(\alpha-\beta C_1)l_1\\ -(\alpha-\beta C_1)n_1+\dfrac{\left(e^{\theta_1(t_2-t_1)}-1\right)(\alpha-\beta C_1)O_1}{\theta_1}\end{array}}{6T}\right\}\\ +4\left\{\dfrac{\begin{array}{c}-kp(\alpha-\beta C_1)+d(\beta C_1-\alpha)-(1-k)(\alpha-\beta C_1)l_2\\ -(\alpha-\beta C_1)n_2+\dfrac{\left(e^{\theta_2(t_2-t_1)}-1\right)(\alpha-\beta C_1)O_2}{\theta_2}\end{array}}{T}\right\}\\ +\left\{\dfrac{\begin{array}{c}-kp(\alpha-\beta C_1)+d(\beta C_1-\alpha)-(1-k)(\alpha-\beta C_1)l_3\\ -(\alpha-\beta C_1)n_3+\dfrac{\left(e^{\theta_3(t_2-t_1)}-1\right)(\alpha-\beta C_1)O_3}{\theta_3}\end{array}}{T}\right\}\end{array}\right]=0$$

Provided it satisfies the equations

$$\frac{\partial^2\widetilde{TC}_G(t_1,t_2)}{\partial t_1^2}>0\ ,\ \frac{\partial^2\widetilde{TC}_G(t_1,t_2)}{\partial t_2^2}>0\quad\text{and}$$

$$\left(\frac{\partial^2\widetilde{TC}_G(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2\widetilde{TC}_G(t_1,t_2)}{\partial t_2^2}\right)-\left(\frac{\partial^2\widetilde{TC}_G(t_1,t_2)}{\partial t_1^2\partial t_2^2}\right)^2>0 \tag{22}$$

## NUMERICAL EXAMPLE

To illustrate the result of the proposed model, let us consider an inventory system with the following parametric values.

*Crisp Model*

$\alpha = 60$ units, $\beta = 0.5$, $C_1 =$ \$ 30/unit/day, $k = 0.7$, $C_2 =$ \$ 10/unit/day, $C_3 =$ \$ 16/unit/day,

$p =$ \$ 15/unit/day, $\theta = 0.006$, $W = 100$ units, $d =$ \$ 16/unit/day, $\tilde{h}_r =$ \$ 0.07/unit,

$\tilde{h}_o =$ \$ 0.06/unit, T=365 days. The values of different parameters considered here are realistic, though these are not taken from any case study. Corresponding to these input values, the optimum value of $t_1 =$ 47.4072 days, $t_2 = 319.925$ days and the minimum value of $TC(t_1, t_2) =$ \$ 2022.03. To show the convexity of cost function $TC(t_1, t_2)$ we plot a 3D graph. A three dimensional graph is shown in the Fig.2.

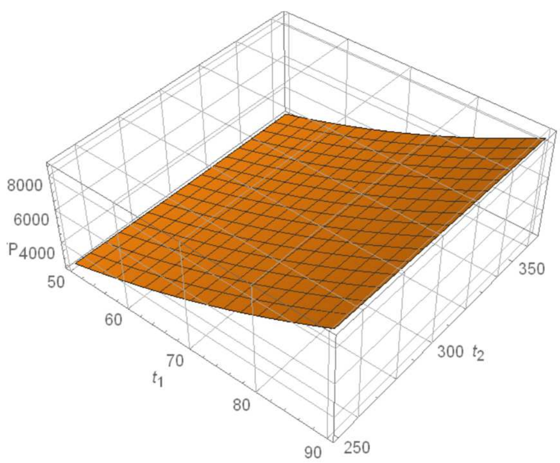

Fig. 2. The convexity of cost function

*Fuzzy Model*

Suppose $\tilde{h}_r = (0.06, 0.07, 0.08)$, $\tilde{h}_o = (0.05, 0.06, 0.07)$, $\tilde{\theta} = (0.005, 0.006, 0.007)$, $\tilde{C}_2 = (8, 10, 12)$, and $\tilde{C}_3 = (14, 16, 18)$ be consider as triangular fuzzy numbers and $\alpha = 60$ units, $\beta = 0.5$, $C_1 =$ \$ 30/unit, $k = 0.7$, $p =$ \$ 15/unit, $W = 100$ units, $d =$ \$ 16/unit, T= 365 days. Then the fuzzy total average cost can be determined by the Graded Mean Representation (GMIR) Method is $\widetilde{TC}_G(t_1, t_2)=$ \$ 2002.17 with optimum value of $t_1 = 47.1471$days, $t_2 = 321.7036$ days.

## SENSITIVITY ANALYSIS

A sensitivity analysis is carried out to study the effect of changes in the system parameters $\tilde{h}_r$, $\tilde{h}_o$, $\tilde{C}_2$, $\tilde{C}_3$, $\tilde{\theta}$. We use Mathematica11.1 software for calculation of the total cost function.

1. When $\tilde{h}_r$, $\tilde{h}_o$, $\tilde{C}_2$, $\tilde{C}_3$, $\tilde{\theta}$ are all triangular fuzzy numbers, then optimum value of $t_1 = 47.1471$days, $t_2 = 321.7036$ days with minimum total cost is $\widetilde{TC}_G(t_1, t_2)=$ \$ 2002.17.
2. When $\tilde{h}_r$, $\tilde{C}_2$, $\tilde{C}_3$, $\tilde{\theta}$ are triangular fuzzy numbers, then optimum value of $t_1 = 47.1382$days, $t_2 = 319.9916$ days with minimum total cost is $\widetilde{TC}_G(t_1, t_2)=$ \$ 2005.9516
3. When $\tilde{C}_2$, $\tilde{C}_3$, $\tilde{\theta}$ are triangular fuzzy numbers, then optimum value of $t_1 = 47.14025$ days, $t_2 = 319.9916$ days with minimum total cost is $\widetilde{TC}_G(t_1, t_2)=$ \$ 2008.2833
4. When $\tilde{C}_2$ *and* $\tilde{C}_3$ are triangular fuzzy numbers, then optimum value of $t_1 = 47.2952$days, $t_2 = 319.7416$ days with minimum total cost is $\widetilde{TC}_G(t_1, t_2)$=\$ 2047.67

In the below tables we analyze the system parameters with different values in fuzzy sense at the same time keeping the other parameters in its original values.

Table 1. Sensitivity analysis on shortage cost parameter $\tilde{C}_2$

| $\tilde{C}_2$ | $t_1$ | $t_2$ | $\widetilde{TC}_G$ |
|---|---|---|---|
| (8, 9, 10) | 43.2842 | 312.5098 | 1712.145 |
| (9, 10, 11) | 47.3906 | 319.8978 | 2025.815 |
| (10, 11, 12) | 51.3978 | 327.1230 | 2362.215 |

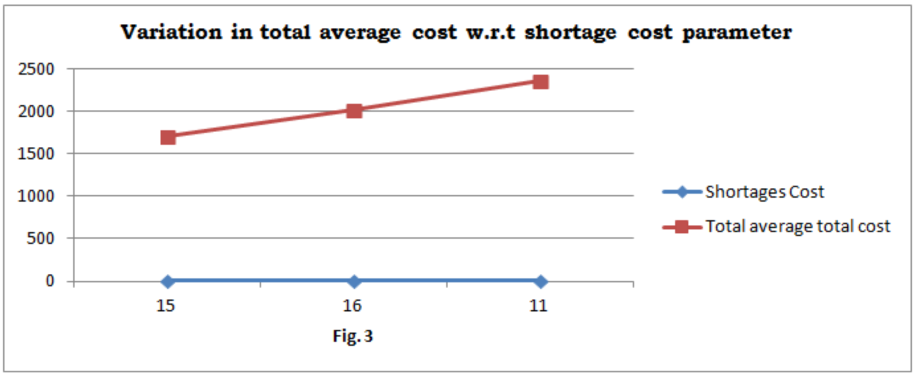


Fig. 3. Sensitivity analysis on shortage cost parameter $\tilde{C}_2$

Table 2. Sensitivity analysis on lost sale cost parameter $\tilde{C}_3$

| $\tilde{C}_3$ | $t_1$ | $t_2$ | $\widetilde{TC}_G$ |
|---|---|---|---|
| (14,15,16) | 46.1846 | 317.7236 | 1925.7750 |
| (15, 16, 17) | 47.4057 | 319.9225 | 2022.3683 |
| (16, 17, 18) | 48.6178 | 322.1065 | 2121.0000 |

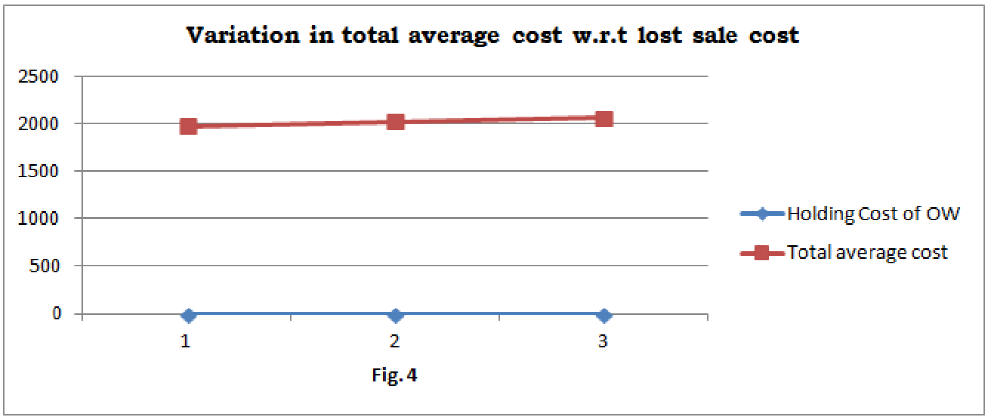


Fig. 4. Sensitivity analysis on lost sale cost parameter $\tilde{C}_3$

Table 3. Sensitivity analysis on deterioration parameter $\tilde{\theta}$

| $\tilde{\theta}$ | $t_1$ | $t_2$ | $\widetilde{TC}_G$ |
|---|---|---|---|
| (0.0056,0.0058,0.006) | 49.0613 | 326.3198 | 2203.9550 |
| (0.0058,0.006,0.0062) | 47.4247 | 319.9738 | 2025.8583 |
| (0.006,0.0062,0.0064) | 45.8939 | 313.9211 | 1871.3216 |

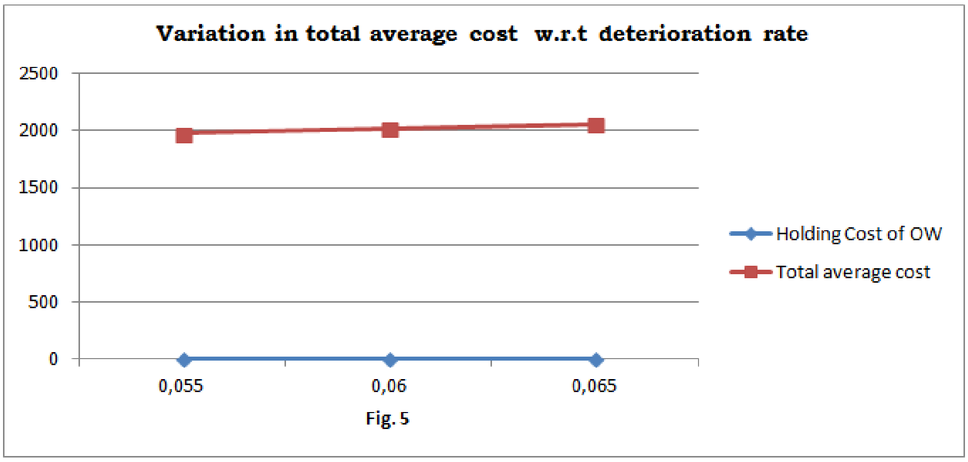


Fig. 5. Sensitivity analysis on deterioration parameter $\tilde{\theta}$

Table 4. Sensitivity analysis on holding cost parameter $\tilde{h}_o$

| $\tilde{h}_o$ | $t_1$ | $t_2$ | $\widetilde{TC}_G$ |
|---|---|---|---|
| (0.05,0.055,0.06) | 47.4411 | 331.9540 | 1978.4387 |
| (0.055,0.06,0.065) | 47.4072 | 320.1111 | 2021.2350 |
| (0.06,0.065,0.07) | 47.3732 | 309.3938 | 2059.2250 |

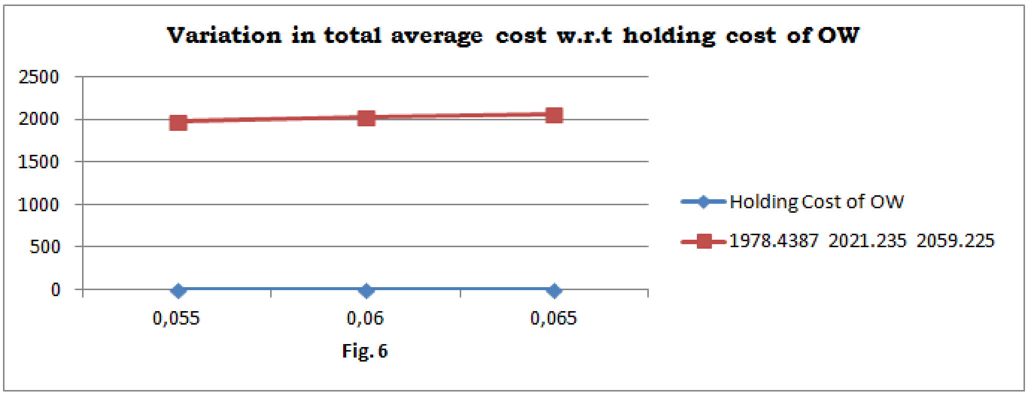


Fig. 6. Sensitivity analysis on holding cost parameter $\tilde{h}_o$

All the above observations can be sum up in fuzzy model as follows:

1) In table 1, fig. 3, if we increase the value of $\tilde{C}_2$ , the optimum value of $t_1$ and $t_2$ increases. By this effect, the total average cost $\widetilde{TC}_G$ increases.

2) In table 2, fig. 4, if we increase the value of $\tilde{C}_3$ , the optimum value of $t_1$ and $t_2$ increases. By this effect, the total average cost $\widetilde{TC}_G$ increases.

3) In table 3, fig. 5, if we increase the value of $\tilde{\theta}$ , the optimum value of $t_1$ and $t_2$ decreases. By this effect, the total average cost $\widetilde{TC}_G$ decreases.

4) In table 4, fig. 6, if we increase the value of $\widetilde{h_o}$ , the optimum value of $t_1$ decreases very slowly as compared to the optimum value of $t_2$. By this effect, the total average cost $\widetilde{TC}_G$ decreases.

## CONCLUSIONS

Most of the researchers worked in two-warehouse inventory modelling have developed only crisp approach. A very few researchers extended their works in fuzzy environment. In this paper, we have developed a two-warehouse inventory model for deteriorating items with price dependent demand and shortages under partially backlogged where the demand rate is a function of selling price. Since deterioration rate, holding cost, shortage cost and lost sale are uncertain, we taken these parameters as triangular fuzzy numbers. The proposed model is discussed both in crisp and fuzzy environment. The optimum result of fuzzy model is defuzzified by Graded Mean Integration Representation (GMIR) method. From this analysis, it is concluded that due to uncertainty nature of the above system parameters, the total average cost decreases in fuzzy model as compared to crisp model. Sensitivity analysis indicates that the total cost function is more sensitive to the changes in deterioration rate. After analyze the result the decision maker can plan for the optimal value for total cost and for other related parameters.

The model can be used for the products like potato, onion, fruits etc. in the countries viz. India, Pakistan, Sri Lanka, Bangladesh etc. as the demand of the food grains increases with time for a fixed time horizon i.e. for a calendar year.

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## REFERENCES

Benkherouf, L., 1997. A deterministic inventory model for deteriorating items with two storage facilities, Int. J. of Production Economics, 48(1), 167-175, http://doi.org/10.1016/S0925-5273(96)00070-9

Bhunia A.K., Jaggi C.K., Sharma A., Sharma R., 2014. A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging, Applied Mathematics and Computation, 232(1), 1125-1137, http://doi.org/10.1016/j.amc.2014.01.115.

Bhunia A.K., Shaikh A.A., Gupta R.K., 2015. A study on two-warehouse partially backlogged deteriorating inventory models under inflation via particle swarm optimization, Int. J. of System Science, 46(8), 1036-1050, http://doi.org/10.1080/00207721.2013.807385

Chakrabarty R., Roy T., Chaudhuri K.S., 2018. A two-warehouse inventory model for deteriorating items with capacity constraints and back-ordering under financial consideration. Int. J. of Appl. and Comp. Math., 4:58, http://doi.org/10.1007/s40819-018-0490-1

Goswami A., Chaudhuri K.S.,1992, An economic order quantity model for items with two levels of storage for a linear trend in demand, J. of Operational Research Society, 43(2),155-167, http://doi.org/10.1057/jors.1992.20

Mandal P., Giri B.C.,2018. A two-warehouse integrated inventory model with imperfect production process under stock dependent demand quantity discount offer', Int. J. of Systems Science: Operations & Logistics, 6(1), 15-26. http://doi.org/10.1080/23302674.2017.1335806

Hartely V.R., 1976. Operations Research-A Managerial Emphasis, Good Year Publishing Company, Santa Monica, California, 315-317.

Huang Y.F., 2006. An inventory model under two levels of trade credit and limited storage space derived without derivatives, Appl. Math. Model, 30(5), 418-436, http://doi.org/10.1016/j.apm.2005.05.009

Jaggi C.K., Cardenas-Barron L.E., Tiwari S., Shafi A.A., 2017. Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments, Scientia Iranica,157(2), 344-356.

Jaggi C.K., Pareek S., Verma P., Sharma R., 2013. Ordering policy for deteriorating items in a two-warehouse environment with partial backlogging, Int. J. of Logistics and Systems Management, 16(1),16-40, http://doi.org/10.1504/IJLSM.2013.055560

Lee C.C., Hsu S.L., 2009. A two-warehouse production model for deteriorating inventtory items with time dependent demands, European J. of Oper. Res., 194(3), 700-710, http://doi.org/10.1016/j.ejor.2007.12.034

Liang Y., Zhou F., 2011. A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payments, Applied Mathematical Modelling, 35(1), 2221-2231, http://doi.org/10.1016/j.apm.2010.11.014

Panda G.C., Khan M.A.A., Shaikh A.A., 2019, A credit policy approach in a two-warehouse inventory model for deteriorating items with price and stock dependent demand under partial backlogging, J. of Industrial Engineering, 15(1), 147-170, http://doi.org/10.1007/s40092-018-0269-3

Roy S.K., Pervin M., Weber G.W., 2018. A two-warehouse probabilistic model with price discount on backorders under two levels of trade credit policy', J. of Industrial Management Optimization, 658-662, http://doi.org/10.3934/jimo.2018167.

Sarma K.V., 1983. A deterministic inventory model with two levels of storage and optimum release rule', Opsearch, 20(1), 175-180.

Sheikh S.R., Patel R., 2017. Two-warehouse inventory model with different deterioration rates under time dependent demand and shortages, Global J. of Pure & Applied Mathematics, 13(8), 3951-3960.

Yang H.L., Chang C.T., 2013. A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation, Applied Mathematical Modelling, 37(1), 2717-2726.

Zadeh L.A., 1965. Fuzzy Set, Information Control, 8(1), 338-353.

Zadeh L.A.. Bellman R.E., 1970. Decision making in a fuzzy environment, Management Science, 17(1), 140-164.

# DWUMAGAZYNOWY ROZMYTY MODEL ZAPASÓW PSUJĄCYCH SIĘ O ZAPOTRZEBOWANIU ZALEŻNYM OD CENY ORAZ UWZGLĘDNIAJĄCY BRAKI

**STRESZCZENIE**. **Wstęp:** W pracy zaprezentowano rozmyty model układu dwumagazynowego, składającego się z własnego magazynu (OW) oraz magazyny wynajmowanego (RW) dla asortymentów podlegających psuciu oraz o popycie zależnym od ceny przy dopuszczenia częściowych braków. Ze względu na ograniczoną powierzchnię własną magazynu, dostawca był zmuszony wynająć drugi magazyn w celu magazynowania nadwyżki. Koszt magazynu wynajmowanego jest wyższy niż koszt magazynu własnego. W pracy uwzględniono koszt utrzymywania obiektu, współczynnik psucia, koszt ubytków oraz koszt utraty sprzedaży jak liczby rozmyte.
**Metody:** W celu odwrócenia rozmycia funkcji całkowitego kosztu użyto metody Graded Mean Integration Representation. Otrzymane wyniki porównano z modelem Crisp przy pomocy przykładu liczbowego. Następnie wykonano analizę wrażliwości zmieniają jeden z parametrów przy utrzymaniu niezmienionych pozostałych.
**Wyniki i wnioski**: Wykazano, że wyniki uzyskane przy zastosowaniu metody Graded Mean Integration Representation są dokładniejsze aniżeli przy zastosowaniu modelu Crisp.

**Słowa kluczowe:** zapas, system dwwumagazynowy, zepsucie, braki, trójkątna liczba rozmyta, metoda Graded Mean Integration Representation

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