



ADAPTIVE DECISION-MAKING STRATEGY FOR SUPPLY CHAIN SYSTEMS UNDER STOCHASTIC DISRUPTIONS

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ABSTRACT. Background: Supply chain management is becoming more complex and essential with the development of the economy and globalization. Due to several interrelated and integrated logistical components, today's global supply chains are typically nonlinear dynamical systems that may show unpredictable, chaotic, or counterintuitive behaviors. In a volatile business environment, a company must integrate a decision-making strategy to achieve its strategic goals. Digitizing any business can keep up with supply chains that have become increasingly global and complex.

Methods: Digital transformation has been rapidly adopted across supply chain networks. A three-echelon supply network has been formulated in discrete time domains for exploring the complex behavior of the dynamical system. The discrete-time models fit more naturally to describe supply chain activities. This paper presents the adaptive management strategy to control the dynamic supply chain systems under uncertainty. The adaptive law is implemented based on the gradient descent method so that it can readily update the control gains of the decision-making strategy. The efficient management strategy helps policymakers implement a decision-support system more precisely and timely.

Results: The paper aims to implement the PID controller with adaptation law in supply chain management's chaotic suppression and synchronization problems under stochastic events. Numerical simulations are presented to evaluate the validity of the proposed algorithms for the operations management of dynamic supply chain networks. The proposed adaptive control strategy provides superior performance and accuracy over classical control strategies. The decision-making algorithms ensuring business profitability are realized by an adaptive management strategy to cope with market disruptions.

Conclusions: Disruptions like customer demand and market conditions impact on the multi-echelon supply chain system. A novel adaptive management strategy is presented to regulate uncertain supply chain systems against market disruptions. The control policy effectively utilizes chaos suppression and synchronization schemes to manage complex supply chain networks. The proposed management solutions will help logistics providers prepare for the future and gain a competitive advantage guaranteeing business resilience and sustainability against a volatile market.

Keywords: Supply chain systems, stochastic process, business profitability, adaptive decision-making strategy

INTRODUCTION

From the present perspective, supply chain management is becoming more complex and essential with the development of the economy and globalization. Due to several interrelated and integrated logistical components, today's global supply chains are typically nonlinear dynamic systems that may show unpredictable, chaotic, or counterintuitive behaviors. A company must

integrate a decision-making strategy to achieve its goals in a volatile business environment. Digitizing any business can keep up with supply chains that have become increasingly global and complex. Digital transformation has been rapidly adopted across supply chain networks. Controlling and managing the supply chain system is more necessary for digital transformation. The control theory can help assist in more effective decisions and improve operational performance. For more than a

decade, many researchers have studied the issues of supply chain design, analysis, modeling, and planning with a nonlinear framework. Most works deal with supply chain systems with continuous-time models, and just a few papers analyzed supply chain systems with discrete-time models. These discrete-time models fit more naturally to describe supply chain activities. In supply chain systems, some imponderable factors might result in nonlinearity and chaotic activities (Kocamaz et al., 2016). The complexity of uncertainty has become the norm in real supply chains. The chaotic supply chain system is one of many supply chain systems becoming the subject of analysis and research. Although chaotic behavior is often considered an undesirable phenomenon, it can provide beneficial features to describe complex nonlinear dynamics of the systems. Chaotic behaviors in the supply chain networks are sometimes caused by sudden changes in demands, disrupted by transportation, weather, disasters, pandemics, and more uncertain factors. The global pandemic and wars have recently posed challenges to supply chains and worldwide logistics, triggering new research areas in supply chain resilience (Ivanov, 2022). To propose the mathematical model for complex supply chain systems, numerous studies have been done on various chaotic systems like Chua, Lorenz, Sprott, Jerk, Lu, etc. Lorenz studied chaos mathematically for the first time in 1963. The chaotic supply chain modeling with the bullwhip effect was introduced (Lei et al., 2006). A three-echelon supply chain with bifurcation analysis and synchronization problems has been described by Anne et al. (2009). Xu et al. (2021) introduced a system dynamics method to manage a chaotic supply chain based on adaptive sliding mode control. Cuong et al. (2021) analyzed a production–distribution model in the nonlinear supply chain system using the adaptive sliding mode controller. Many methods are proposed for modeling the supply chain systems. The system dynamics can be analyzed in continuous-time form (Anne et al., 2009; Ardakani et al., 2020; Cuong et al., 2021; Ghadimi & Aouam, 2021; Lei et al., 2006), while some researchers analyze the system in discrete-time form (Tempelmeier, 2006). In aspects of the relationship between productivity and distribution, some articles discuss a mixed-integer programming model for a multi-period, multi-product supply chain considering conflicting economic and social

responsibility objectives (Ardakani et al., 2020); optimizing the production capacity and safety stocks in a serial production–distribution system providing multiple products under a guaranteed service approach (Ghadimi & Aouam, 2021); analyzing and managing production–distribution in a nonlinear supply chain model using sliding mode control theory (Cuong et al., 2021). A multi-echelon nonlinear framework, which can be transformed into the Lorenz equation as a chaotic demonstration, has been presented to describe the supply chains considering the production of the manufacturer, distributor storage, retailer transportation, safety stock, and customer satisfaction (Lei et al., 2006; Anne et al., 2009; Xu et al., 2021). Numerous chaotic cases exist in complex supply chain networks, and economic and business models. A small change can be amplified to have a large effect on the system with a highly sensitive dependence on initial conditions. Chaotic phenomena contribute to exploring short-term changes in demand as the disruptive behavior experienced within the supply chain networks. For the decision-making strategy on chaotic suppression and synchronization problems, the control theory might provide sufficient mathematical tools to analyze, design, and simulate the supply chain management systems based on a system dynamics approach (Sarimveis et al., 2008). The main goals of chaotic system control are to realize the closed-loop supply chain systems for removing chaotic behaviors and synchronizing sudden changes in market demands. Several control theories have been proposed in regulating chaotic systems: linear feedback control (Kocamaz & Uyaroğlu, 2014) was considered to regulate continuous time Rucklidge chaotic system; an adaptive sliding mode controller (Xu et al., 2021; Cuong et al., 2021) was realized in managing chaotic supply chains in a stochastic environment; intelligent control and synchronization of chaotic supply chains (Kocamaz et al., 2016) were implemented using adaptive neuro-fuzzy inference system; and robust controllers (Govindan & Cheng 2018; Gholami et al., 2019; Zhang & Cui, 2021) were proposed for dynamical systems, ensuring robustness against disturbances. Among the control theories, the PID controller is considered a robust control technique for nonlinear dynamical systems (Amir et al., 2020). This controller is still widely used because of its structural simplicity, reputation, robustness, and

easy implementation. Its form is built based on the current tracking error, the sum of recent errors, and the derivative of the error. This paper aims to implement the PID controller with an adaptation law in the supply chain management's chaotic suppression and synchronization problems under stochastic events. An adaptation law will be designed based on errors between the actual system affected by disturbances and the system in ideal condition. The adaptation law can be integrated with other controllers such as sliding mode control (Chang & Yan, 2005); based on system error (Tsai et al., 2017; Zhang & Cui, 2021), and so on. The structure of this article has 5 sections. Section 2 introduces the supply chain system model with a local stability analysis. This is a discrete-time model using the Lorenz equation. The adaptive PID control synthesis with stability analysis is described in Section 3. Section 4 presents the results of numerical simulations to demonstrate the effectiveness of the designed control strategy. Finally, conclusions are made in Section 5.

MATHEMATICAL MODEL OF NONLINEAR SUPPLY CHAIN SYSTEMS

In this study, a multi-stage or multi-echelon supply chain system consists of multiple tiers, such as the manufacturer, the distributor, the retailer, and the customer. For dynamic modeling, an essential requirement is a clear understanding of the relationship among many elements in the supply chain systems. The dynamic model creates three typical processes: the manufacturer producing, the distributor handling, and the retailer shipping products to customers. The complex model incorporates different flows of activities, mechanisms, and functions. The generic supply chain model is shown in Fig. 1.

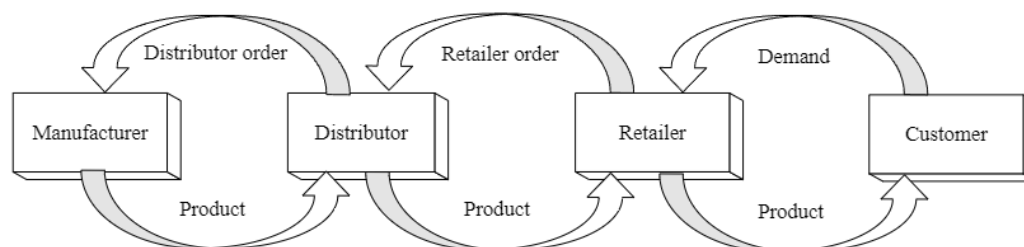


Fig. 1. Schematic diagram of the generic supply chain model.

Excellent supply chain management is crucial to profitability and maximizing customer satisfaction for any business. While typical management models are effectively described in a discrete-time domain, the supply chain dynamics are formulated and defined at each instance. It is noted that a fundamental variable is an inventory (or stock) in each echelon of the supply chain network. For the mathematical model, the following notations are used in this research (Lei et al., 2006):

i Time period.

x_i The quantity that the retailer of the products sends to the customer in the current period.

y_i The quantity that the distributor of the products handles in the current period.

z_i The quantity of products the manufacturer produces in the current period.

γ The delivery efficiency of the distributor.

σ The efficiency of the customer demand.

ε The efficiency of the retailer sending an order to the distributor.

θ The safety coefficient of the manufacturer.

d The stochastic disturbance.

u The control action in the decision-making.

In reality, the demand information transferred between each chain is delayed, leading to lead time in the supply chains. It means that the current order that the distributor received in the current period is the demand order of the retailer in the previous period. The quantity of the products that the retailer sends to the customer, x_i , depends on the number of products delivered from the distributor to the retailer and the number of satisfied customers in the previous period. This relation is given as follows:

$$x_i = \gamma y_{i-1} - \sigma x_{i-1} \quad (1)$$

The quantity of the products the distributor handles, y_i , is affected by the manufacturer's production and the retailer's order ($x_{i-1}z_{i-1}$). It also depends on the number of products the retailer sends to customers. The quantity of products the distributor sends to the retailer is given below:

$$y_i = \varepsilon x_{i-1} - x_{i-1}z_{i-1} \quad (2)$$

The last echelon in the system is the manufacturer, in which z_{i-1} denotes the number of products the manufacturer produces in the current period. To make a production decision exactly, the manufacturer has to get information from both the retailer and distributor ($x_{i-1}y_{i-1}$). In the real market, the manufacturer often makes more than demand, and it requires the safety coefficient based on the previous period (θz_{i-1}). The quantity of products the manufacturer produces is given below,

$$z_i = x_{i-1} \cdot y_{i-1} + \theta z_{i-1} \quad (3)$$

where θ is the relative number, which is the supplier's capacity. This sometimes makes the quantity produced more or less than the previous period's demand or quite different from the demand they received from the market.

However, in reality, the supply chain system is affected by many stochastic factors. The real disturbances are introduced into the system as stochastic signals and added to the model, which can be used to describe actual market conditions. Then, the complete model is written as follows:

$$\begin{aligned} x_i &= \gamma y_{i-1} - \sigma x_{i-1} + d_{x_{i-1}} \\ y_i &= \varepsilon x_{i-1} - x_{i-1}z_{i-1} + d_{y_{i-1}} \\ z_i &= x_{i-1} \cdot y_{i-1} + \theta z_{i-1} + d_{z_{i-1}} \end{aligned} \quad (4)$$

where the disturbances ($d_{x_{i-1}}$, $d_{y_{i-1}}$, and $d_{z_{i-1}}$) are given by the Wiener process or Brownian process and the widely used random or stochastic process. Brownian motion is frequently used to model supply chain systems and finance problems. It has been noted that stochastic behavior occurs in a deterministic manner in supply chain systems. The equilibrium analysis of the ideal system without disturbances is presented to explore the local behaviors of the nonlinear dynamical model. The specific parameters of the system are chosen as $(\gamma, \sigma, \varepsilon, \theta) = (10; 5; 15; -0.2)$ for numerical analysis. The three equilibrium points are determined by $E_1 = (0; 0; 0)$, $E_2 = (1.267; 0.76; 0.802)$ and $E_3 = (-1.267; -0.76; 0.802)$. The Routh–Hurwitz criterion checks the local stability of near equilibrium points. Using the Jacobian matrix, the eigenvalues (λ_{E_i}) of equilibrium points (E_1, E_2 , and E_3) are obtained as follows:

$$\begin{aligned} \lambda_{E_1} &= (-15; 10; -0.2) \\ \lambda_{E_2} &= \lambda_{E_3} \\ &= (-14.6583; 9.5351; -0.0768) \end{aligned} \quad (5)$$

where the eigenvalues identify unstable equilibria of a set of difference equations. As illustrated in Fig. 2, it is unstable if any eigenvalue at the equilibrium point has a positive real value. It is worth noting that this equilibrium analysis provides only the local stability of the ideal system.

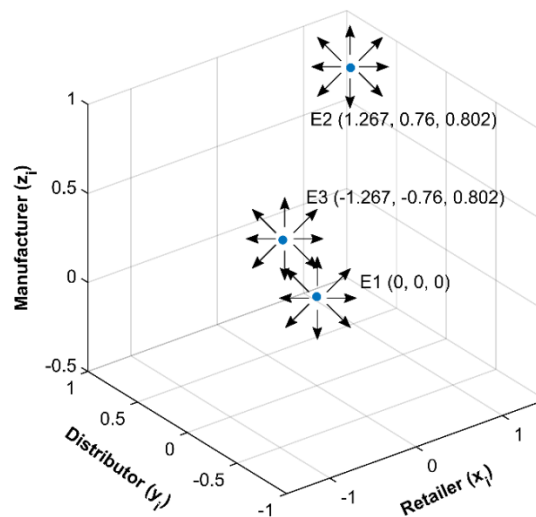
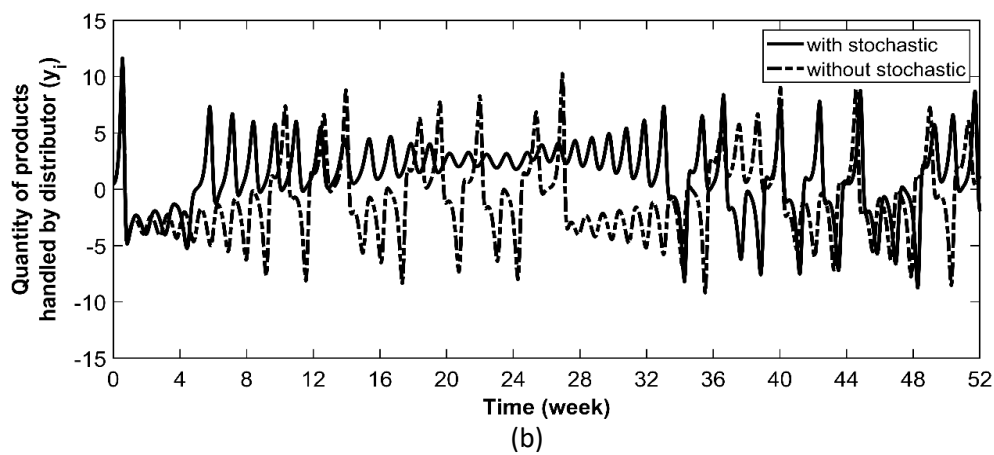
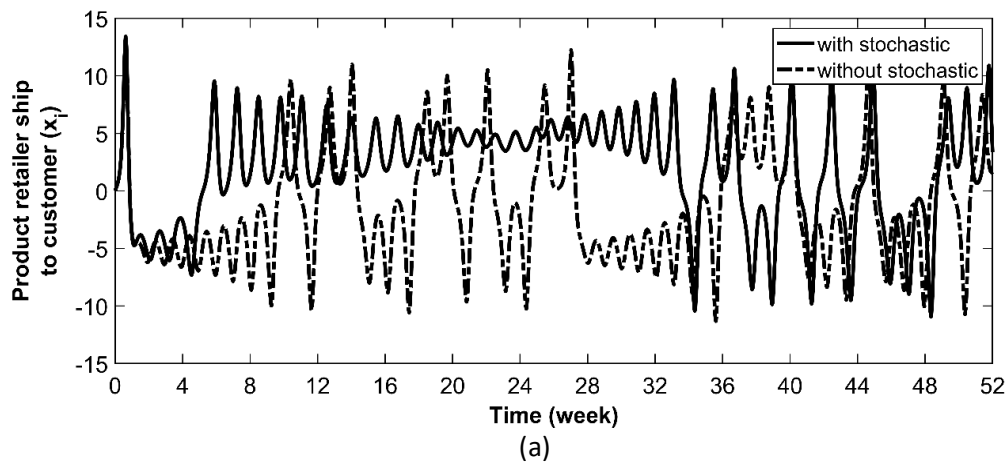


Fig. 2. Equilibrium analysis for the ideal system

Next, Fig. 3 shows the system response characteristics in the given periods (52 weeks of the stock level), including the ideal and uncertain systems with stochastic effects.



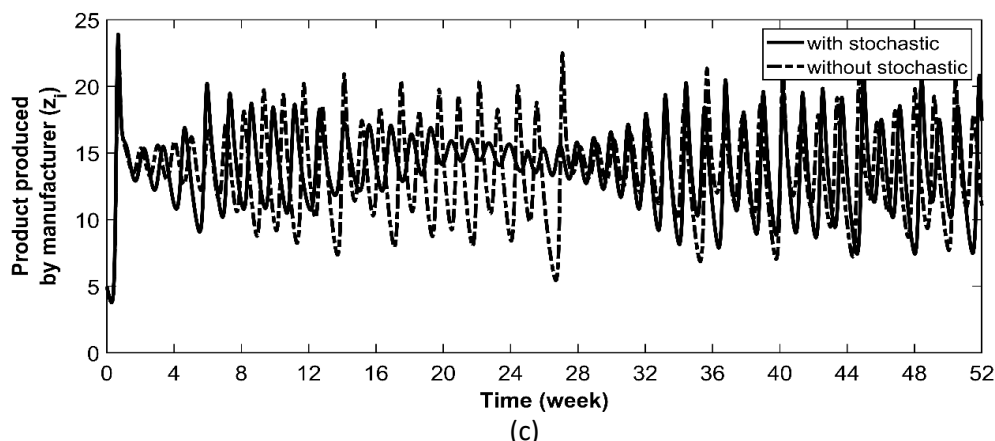


Fig. 3. Time responses of the supply chain system (in 52 weeks): (a) product retailer ships to the customer, (b) quantity of products handled by the distributor, and (c) product produced by the manufacturer.

DESIGN OF ADAPTIVE MANAGEMENT STRATEGY

The key objective in robust supply chain management is to satisfy customer demand while keeping optimal inventory (stock) levels in each echelon of the network against market disruptions. The robustness is related to the ability of the supply chains to resist changes under various (external or internal) disturbances. Building a robust management strategy gains more and more significance in the face of volatile markets. There is no single best way to realize an efficient supply chain strategy. In this study, a management strategy is designed based on the robust adaptive controller for making decisions quickly and efficiently. It is noted that efficient decision-making is essential for business success against disruptions. At each stage, the control input is the product in the system's transportation. For the closed-loop system design, a specific control input signal is introduced as u_i in the system (4). Then, the complete system is written in a compact form incorporating disturbance and control input as follows:

$$\begin{aligned} x_i &= \gamma y_{i-1} - \sigma x_{i-1} + d_{xi} + u_{xi} \\ y_i &= \varepsilon x_{i-1} - x_{i-1} z_{i-1} + d_{yi} + u_{yi} \\ z_i &= x_{i-1} y_{i-1} + \theta z_{i-1} + d_{zi} + u_{zi} \end{aligned} \quad (6)$$

As seen in Figs. 2 and 3, the chaotic attractor is observed when the system parameter

values are chosen explicitly as follows: $(\gamma, \sigma, \varepsilon, \theta) = (10; 5; 15; -0.2)$, in which the external disturbances (d_{xi} , d_{yi} and d_{zi}) are described as stochastic processes. This model illustrates chaotic and complex dynamical behaviors within the framework of nonlinear dynamical systems (Xu et al., 2021). As mentioned, numerous chaotic phenomena exist in complex supply chain networks, illustrating sensitivity to initial conditions. Adaptive PID control with an adaptation law will be employed to change the dynamics of the supply chain model. As the controlled supply chain network becomes more extensive and complex, the supervisory controller will be additionally realized in the control system as the model-based strategy for discrete event models, which can be considered the boundary of the control system. Then the control input is given by a two-stage control mechanism,

$$u = \eta u_{pid} + \mu u_s \quad (7)$$

where η and μ are the effect factors of each control element; u_{pid} is the PID controller signal; u_s is the supervisory controller, which will be activated only when the state of the system exceeds some bounds and guarantees the stability of the system. Based on the gradient method, a proper adaptive law is designed to minimize the tracking error by updating control gains. Fig. 4 shows the general structure of the proposed adaptive control synthesis.

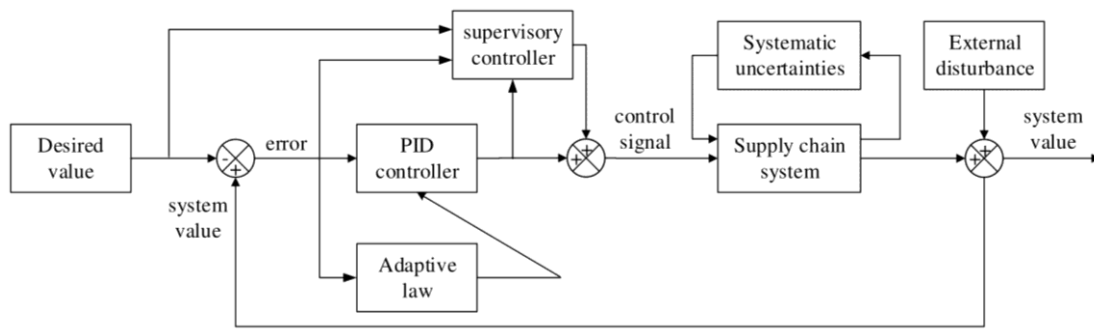


Fig. 4. Block diagram of the proposed adaptive control strategy

This two-state control scheme is intended to achieve better performance and strong robustness under stochastic events for making excellent and timely management decisions. In supply chain management, if the chaotic phenomena at stock levels across the stages cause unwanted problems, an active control scheme becomes necessary to eliminate undesirable dynamical behaviors. Furthermore, the synchronization scheme is essential for ensuring punctual optimal stock level, even if it has chaotic trajectories due to disruptions. Both scenarios are comprehensively discussed to guarantee a practical management strategy.

Dynamic suppression scheme

Since a classical PID controller is a reliable controller with a simple structure, it has been widely used in many applications for several decades (Parnianifard et al., 2020). The discrete form of the controller is generally given as follows:

$$u_{pid_i} = K_p e_i + K_i \sum_{j=0}^i e_j + K_d (e_i - e_{i-1}) \quad (8)$$

The difference in the control signals taking data consecutively is given below:

$$\begin{aligned} \Delta u_i &= u_i - u_{i-1} \\ &= K_p (e_i - e_{i-1}) \\ &\quad + K_i e_i \\ &\quad + K_d (e_i - 2e_{i-1} \\ &\quad + e_{i-2}) \end{aligned} \quad (9)$$

From Eqs. (8) and (9), the control signal u_{pid} is described by,

$$\begin{aligned} u_{pid_i} &= K_p (e_i - e_{i-1}) + K_i e_i \\ &\quad + K_d (e_i - 2e_{i-1} \\ &\quad + e_{i-2}) + u_{pid_i-1} \end{aligned} \quad (10)$$

where e_i , e_{i-1} , and e_{i-2} are the error signals, and K_p , K_i , and K_d specify the proportional, integral, and derivative gains, respectively. The vector of the tracking errors is defined as $e_i = [e_{xi}, e_{yi}, e_{zi}]^T$, and they can be given by

$$\begin{aligned} e_{xi} &= x_r - x_i, e_{yi} = y_r - y_i, \\ \text{and } e_{zi} &= z_r - z_i \end{aligned} \quad (11)$$

where x_r , y_r , and z_r are the desired reference signals for the system. Eq. (6) is rewritten in compact form using matrices and vectors:

$$w_i = Aw_i + f(w_i) + d_i + u_i \quad (12)$$

where $w_i = [x_i, y_i, z_i]^T$, $f(w_i) = [0, -x_i z_i, x_i y_i]^T$, $d_i = [d_{xi}, d_{yi}, d_{zi}]^T$ and $u_i = [u_{xi}, u_{yi}, u_{zi}]^T$.

Using the error vector, $e_i = w_r - w_i$, the control signal at each instance in the system is given by

$$u_{pid} = \begin{bmatrix} u_{pid_x_i} \\ u_{pid_y_i} \\ u_{pid_z_i} \end{bmatrix} = \begin{bmatrix} K_{px}(e_{x_i} - e_{x_{i-1}}) + K_{ix}e_{x_i} + K_{dx}(e_{x_i} - 2e_{x_{i-1}} + e_{x_{i-2}}) + u_{pid_x_{i-1}} \\ K_{py}(e_{y_i} - e_{y_{i-1}}) + K_{iy}e_{y_i} + K_{dy}(e_{y_i} - 2e_{y_{i-1}} + e_{y_{i-2}}) + u_{pid_y_{i-1}} \\ K_{pz}(e_{z_i} - e_{z_{i-1}}) + K_{iz}e_{z_i} + K_{dz}(e_{z_i} - 2e_{z_{i-1}} + e_{z_{i-2}}) + u_{pid_z_{i-1}} \end{bmatrix} \quad (13)$$

To derive the adaption laws for the control gains, the error function is defined below.

$$E_i = \frac{1}{2} e_i^T e_i = \frac{1}{2} (e_{xi}^2 + e_{yi}^2 + e_{zi}^2) \quad (14)$$

The parameters (K_p , K_i , and K_d) can be tuned by using the gradient descent method. The rules to update the controller parameters are

$$K_{Pj}(i+1) = K_{Pj}(i) + \Delta K_{Pj}(i) \quad (15)$$

$$K_{Ij}(i+1) = K_{Ij}(i) + \Delta K_{Ij}(i) \quad (16)$$

$$K_{Dj}(i+1) = K_{Dj}(i) + \Delta K_{Dj}(i) \quad (17)$$

where the subscript is specified by $j = \{x, y, z\}$. Using the chain rule, the following equations are obtained:

$$\begin{aligned} \Delta K_{Pj} &= -\psi_{Pj} \frac{\partial E_i}{\partial K_{Pj}} \\ &= -\psi_{Pj} \frac{\partial E_i}{\partial e_{ji}} \frac{\partial e_{ji}}{\partial j_i} \frac{\partial j_i}{\partial u_{pid_j}} \frac{\partial u_{pid_j}}{\partial K_{Pj}} \end{aligned} \quad (18)$$

$$\begin{aligned} &= \psi_{Pj} e_{ji} \cdot (e_{ji} - e_{ji-1}) \\ \Delta K_{Ij} &= -\psi_{Ij} \frac{\partial E_i}{\partial K_{Ij}} \\ &= -\psi_{Ij} \frac{\partial E_i}{\partial e_{ji}} \frac{\partial e_{ji}}{\partial j_i} \frac{\partial j_i}{\partial u_{pid_j}} \frac{\partial u_{pid_j}}{\partial K_{Ij}} \end{aligned} \quad (19)$$

$$\begin{aligned} &= \psi_{Pj} e_{ji}^2 \\ \Delta K_{Dj} &= -\psi_{Dj} \frac{\partial E_i}{\partial K_{Dj}} \\ &= -\psi_{Dj} \frac{\partial E_i}{\partial e_{ji}} \frac{\partial e_{ji}}{\partial j_i} \frac{\partial j_i}{\partial u_{pid_j}} \frac{\partial u_{pid_j}}{\partial K_{Dj}} \end{aligned} \quad (20)$$

$$= \psi_{Pj} e_{ji} \cdot (e_{ji} - 2e_{ji-1} + e_{ji-2})$$

where ψ_{Pj} , ψ_{Ij} , and ψ_{Dj} are the positive learning rates ($j = \{x, y, z\}$). Next, the model-based approach to supervisory control is realized as follows:

$$u_{si} = -Aw_i - f(w_i) - d_i + \delta_i \quad (21)$$

where $\delta_i = [\delta_{xi}, \delta_{yi}, \delta_{zi}]^T$ is the positive value set. Next, the closed-loop stability of the system will be analyzed using the Lyapunov theory.

Theorem 1: The system trajectories of the controlled supply chain system (6) converge to the set points in finite time if the controller is designed as Eq. (7) and adaptive law as Eqs. (15)–(17).

Proof: First, the Lyapunov energy function is considered as follows:

$$V_{2i} = \frac{1}{2} e_i^T P e_i \quad (22)$$

where P is a positive definite symmetric matrix satisfying the Lyapunov equation:

$$A^T P + P A = -Q \quad (23)$$

and Q is also a positive definite symmetric matrix chosen by the system designer. The difference in function V_2 is defined by

$$\Delta V_{2i+1} = V_{2i+1} - V_{2i} \quad (24)$$

The stable condition is defined as $\Delta V_2 \leq 0$. Eq. (24) is re-written as:

$$\begin{aligned}
 \Delta V_{2i+1} &= \frac{1}{2} e_{i+1}^T P e_{i+1} - \frac{1}{2} e_i^T P e_i = \frac{1}{2} e_{i+1}^T P (w_{i+1} - w_i) - \frac{1}{2} e_i^T P e_i \\
 &= \frac{1}{2} e_{i+1}^T P (A w_i + f(w_i) + d_i + u_i - A w_{i-1} - f(w_{i-1}) - d_{i-1} - u_{i-1}) - \frac{1}{2} e_i^T P e_i \\
 &= -\frac{1}{2} e_{i+1}^T P [A(w_{i-1} - w_i) + (f(w_{i-1}) - f(w_i)) + (d_{i-1} - d_i) + (u_{i-1} - u_i)] \frac{1}{2} e_i^T P e_i \quad (25) \\
 &= -\frac{1}{2} e_{i+1}^T P [A(w_{i-1} - w_i) + (f(w_{i-1}) - f(w_i)) + (d_{i-1} - d_i) + \eta u_{pid_{i-1}} + \mu u_{s_{i-1}} - \eta u_{pid_i} - \mu u_{s_i}] - \frac{1}{2} e_i^T P e_i \\
 &= -\frac{1}{2} e_{i+1}^T P [A(w_{i-1} - w_i) + (f(w_{i-1}) - f(w_i)) + (d_{i-1} - d_i) + \eta u_{pid_{i-1}} \\
 &\quad + \mu(-A w_{i-1} - f(w_{i-1}) - d_{i-1} - \delta_{i-1}) - \eta u_{pid_i} - \mu(-A w_i - f(w_i) - d_i - \delta_i)] - \frac{1}{2} e_i^T P e_i \\
 &= -\frac{1}{2} e_{i+1}^T P [\eta u_{pid_{i-1}} - \eta u_{pid_i} - \delta_{i-1} + \delta_i] - \frac{1}{2} e_i^T P e_i
 \end{aligned}$$

In the formula, some assumptions are made, such as $\delta_{i-1} \leq \eta u_{pid(i-1)}$ and $\delta_i \geq \eta u_{pid(i)}$, so that $\Delta V_2(e_i) \leq 0$ is ensured for the closed-loop stability. The proof is completed. ■

By employing Theorem 1, the proposed control strategy realizes the suppression scheme.

Synchronization scheme

The previous section shows how to design an adaptive management strategy to suppress deterministic chaos. With a synchronization strategy, the business enterprises would produce the exact number of goods necessary to meet actual demand in real-time. It uses a control input to force the system toward the desired value. In this scheme, it is essential to define the ideal reference model of a nonlinear master and slave chaotic system. The control input will drive the chaotic system (slave), affected by disturbances, to follow the ideal supply chain dynamics (master). The ideal reference system equation (master) is given by

$$\begin{aligned}
 x'_{ri} &= \gamma y'_{ri-1} - \sigma x'_{ri-1} \\
 y'_{ri} &= \varepsilon x'_{ri-1} - x'_{ri-1} z'_{ri-1} \\
 z'_{ri} &= x'_{ri-1} \cdot y'_{ri-1} + \theta z'_{ri-1}
 \end{aligned} \quad (26)$$

From Eq. (6), the actual system (slave) with the disturbances and the controller is given by

$$\begin{aligned}
 x'_i &= \gamma y'_{i-1} - \sigma x'_{i-1} + d'_{xi} + u'_{xi} \\
 y'_i &= \varepsilon x'_{i-1} - x'_{i-1} z'_{i-1} + d'_{yi} + u'_{yi} \\
 z'_i &= x'_{i-1} \cdot y'_{i-1} + \theta z'_{i-1} + d'_{zi} + u'_{zi}
 \end{aligned} \quad (27)$$

This formulation deals with the synchronization strategy of two identical supply chains with different initial conditions. By setting $w'_i = [x'_i, y'_i, z'_i]^T$ and $w'_{ri} = [x'_{ri}, y'_{ri}, z'_{ri}]^T$, the error vector of the synchronized two systems is written as follows:

$$e'_i = w'_i - w'_{ri} \quad (28)$$

Based on the control synthesis described before, the proposed control algorithm is designed by

$$u'_i = u'_{pid(i)} + u'_{s(i)} \quad (29)$$

Similarly, the PID controller and supervisory controller are given by

$$\begin{aligned}
 u'_{pid(i)} &= K'_{p(i)} \cdot (e'_i - e'_{i-1}) + K'_{i(i)} e'_i \\
 &\quad + K'_{d(i)} \cdot (e'_i - 2e'_{i-1} + e'_{i-2})
 \end{aligned}$$

$$u'_{s(i)} = -A' w'_i - f'(w_i) - d'_i + \delta'_i$$

Theorem 2: The synchronizing trajectories of the supply chain systems in Eqs. (26)-(27) with an unknown parameter for any initial conditions will be asymptotically stable by employing the adaptive control law described in Eq. (29).

Proof: Since the proof is similar to that of Theorem 1, the detailed proof is omitted for brevity. ■

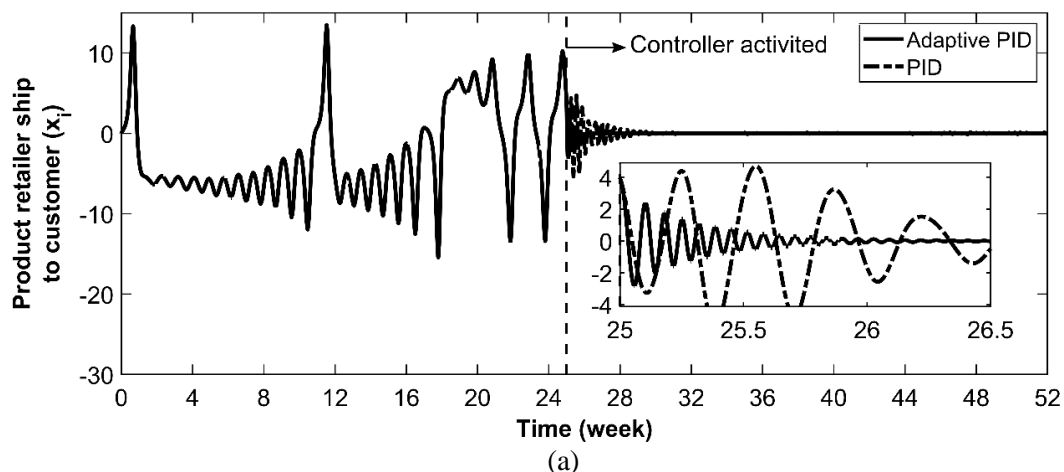
Likewise, by utilizing Theorem 2, the synchronization error signals are asymptotically zero, $\|e'_i\|_2 = 0$ or $w'_i \rightarrow w'_{ri}$ asymptotically. Then the master-slave synchronization scheme is realized by the proposed control strategy.

NUMERICAL SIMULATIONS

In this section, numerical simulations will be carried out to verify the management strategy's performance when applied to the supply chain control system. The decision-makers want to keep the optimal stock level susceptible to oscillations in demand and inventory level as orders pass through the supply chain networks under uncertainty. Sometimes, effective short-term decision-making processes are essential for guaranteeing business sustainability through resilience. Chaotic phenomena contribute to short-term changes in demand as disruptive behavior is experienced within the supply chain networks. Several controllers can be implemented to explore their effects on efficiency and performance for timely and efficient policy making. Two chaotic scenarios are thoroughly discussed to guarantee a practical management strategy.

Chaos suppression policy

First, removing chaos is the key to managing uncertain supply chain networks as a risk management strategy against volatility. The initial control gains are selected as follows: $(K_{px}, K_{ix}, K_{dx}) = (1.5, 0.85, 0.5)$; $(K_{py}, K_{iy}, K_{dy}) = (2, 0.2, 1.5)$; $(K_{pz}, K_{iz}, K_{dz}) = (2.5, 0.3, 2)$; and $(x_r, y_r, z_r) = (0, 0.5, 5)$. For comparison purposes, the system performance with the classical PID controller is shown in Figs. 5~7, where the control parameters are chosen as follows: $(K_{px_r}, K_{ix_r}, K_{dx_r}) = (1, 0.75, 0.5)$; $(K_{py_r}, K_{iy_r}, K_{dy_r}) = (1.5, 0.2, 1)$; and $(K_{pz_r}, K_{iz_r}, K_{dz_r}) = (2, 0.3, 0.75)$. The control performances on the time response, the tracking error, and the control activity are illustrated in Figs. 5, 6, and 7, respectively. The control action starts on the 25th week over 52 weeks. The simulation results show that the proposed adaptive control strategy provides superior performance and accuracy over classical control law. Notably, the control activity (or energy expenditure) required for the adaptive control approach is smaller than the control system with a classical controller. The activity signals are closely related to the control energy and time spent on decision-making. The strategic business policy is a standing plan that provides guidelines for timely and efficient decision-making. Based on the proposed approach, the policymaker can fully control the supply chain systems with less decision-making action.



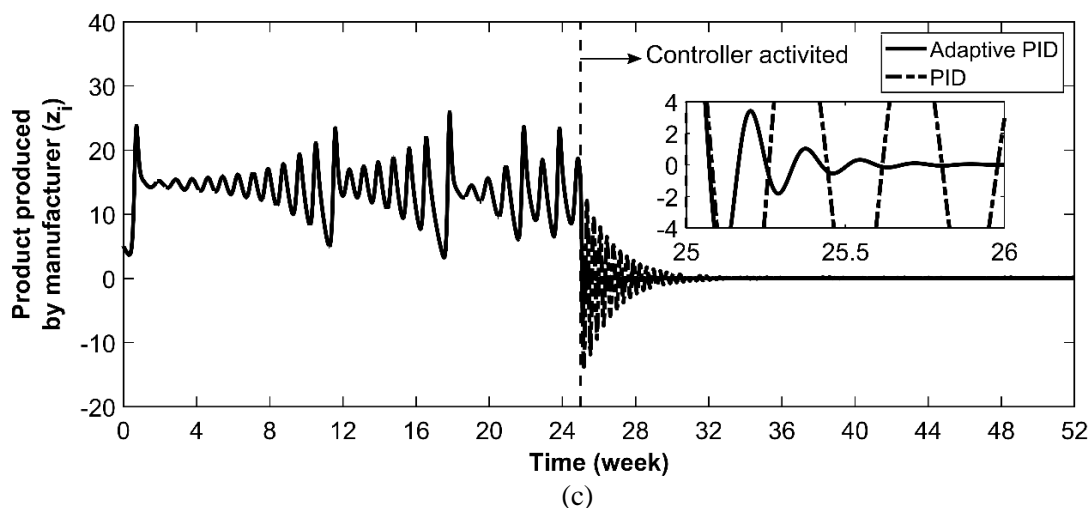
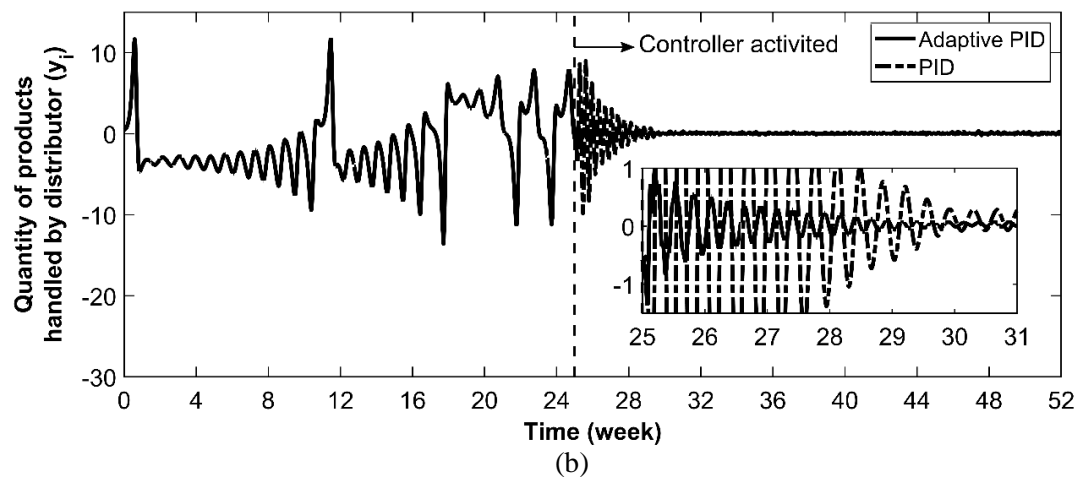
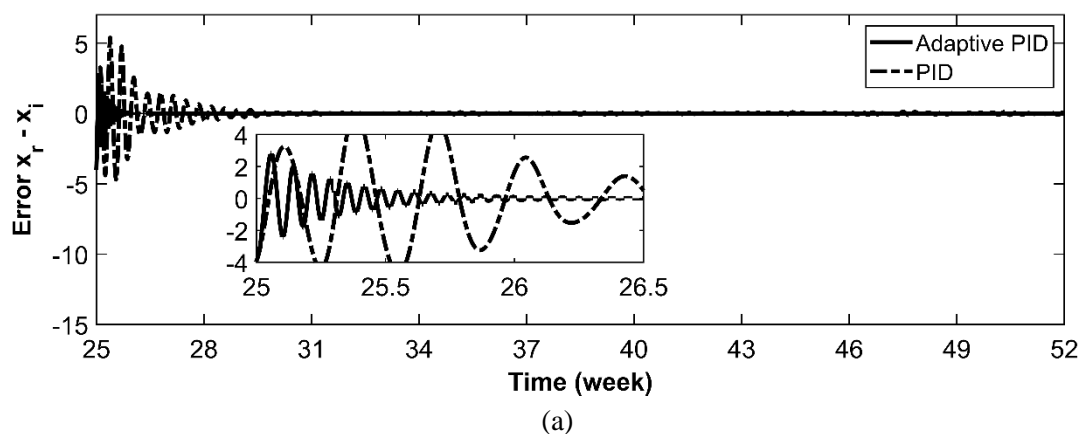
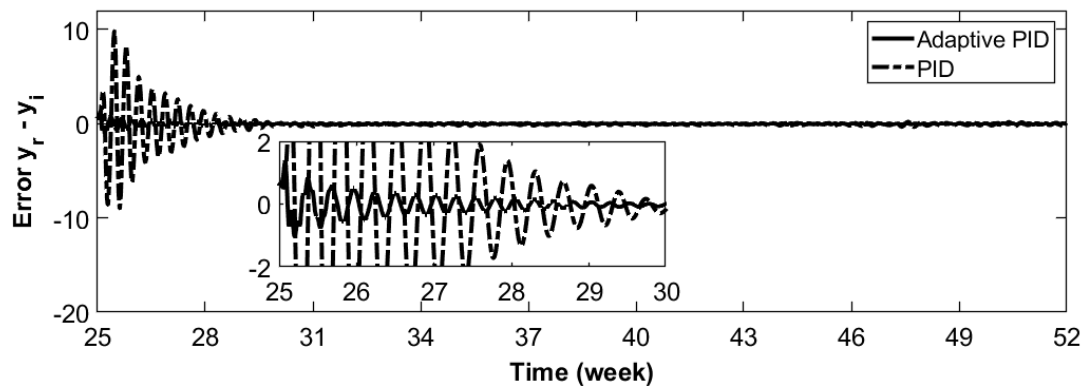
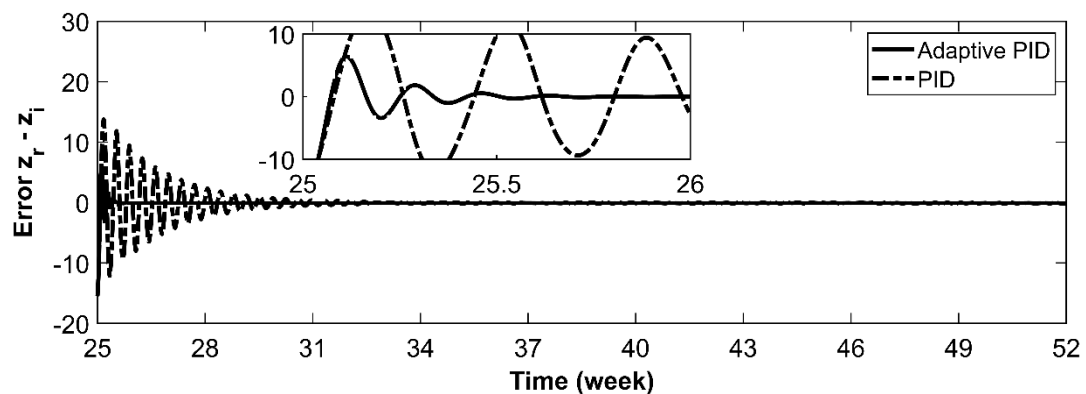


Fig. 5. Time responses of the system with control action at the 25th week: (a) product retailer ships to the customer, (b) quantity of products handled by the distributor, and (c) product produced by the manufacturer.



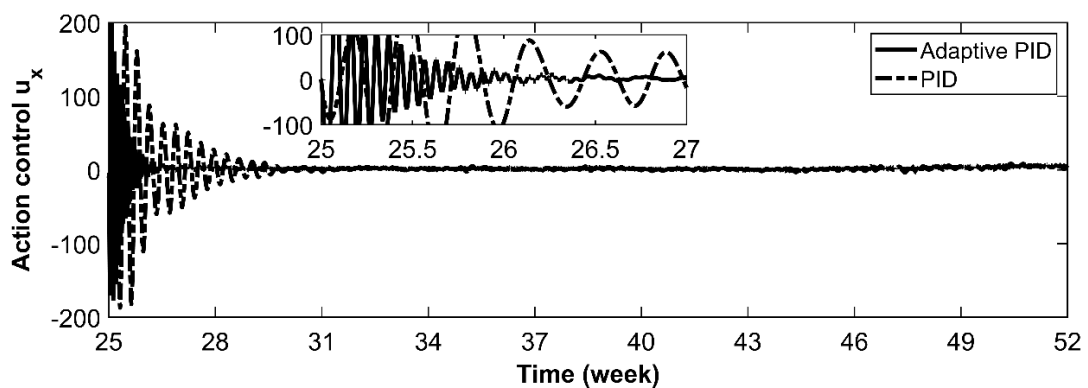


(b)



(c)

Fig. 6. Tracking performances of the system with control action at the 25th week: (a) tracking error of the retailer, (b) tracking error of the distributor, and (c) tracking error of the manufacturer.



(a)

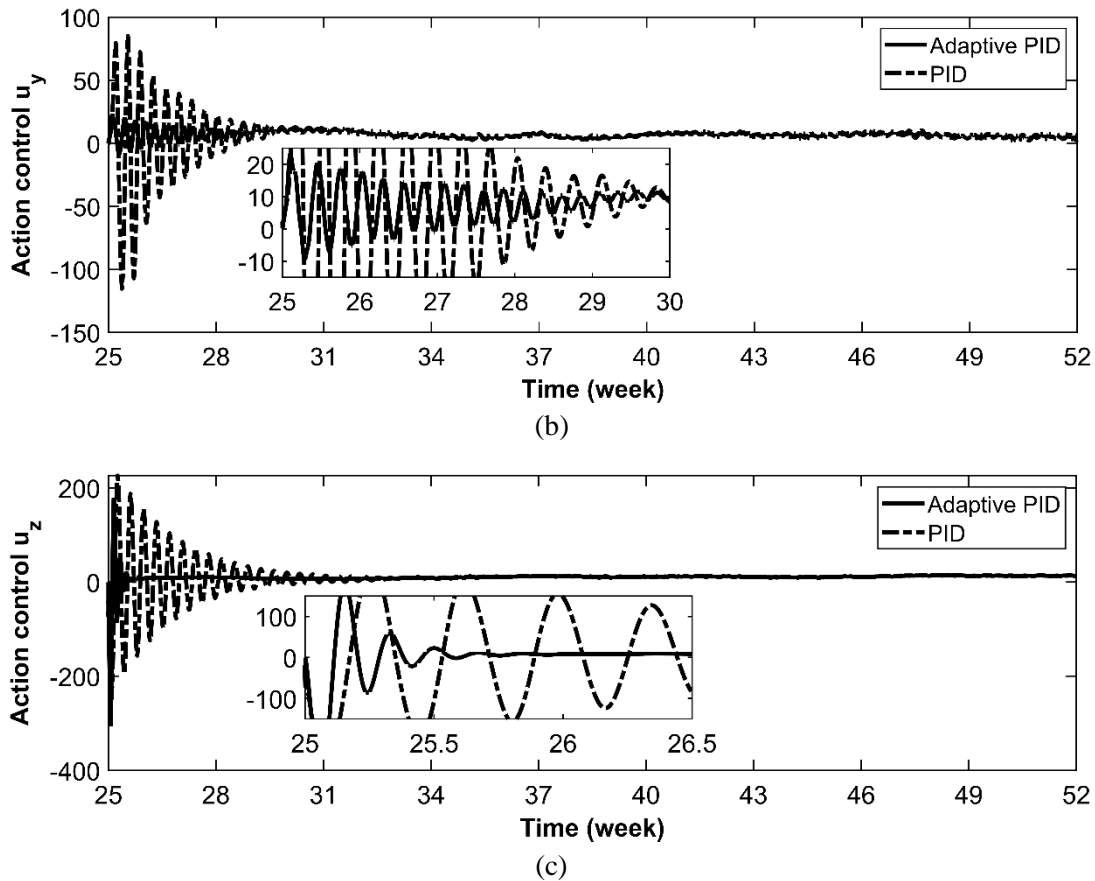
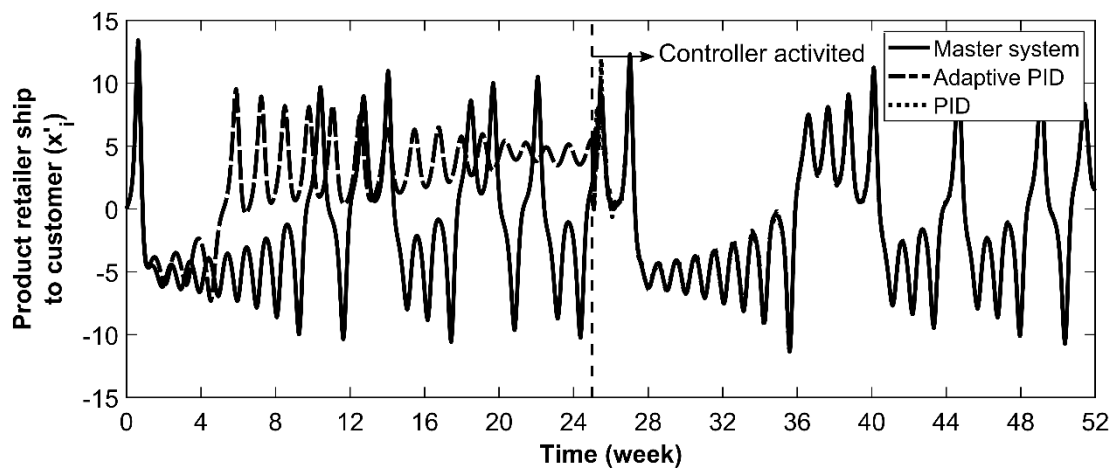


Fig. 7. Control activity signals (or energy expenditure) of the control laws for suppression policy: (a) control action at the retailer, (b) control action at the distributor, and (c) control action at the manufacturer.

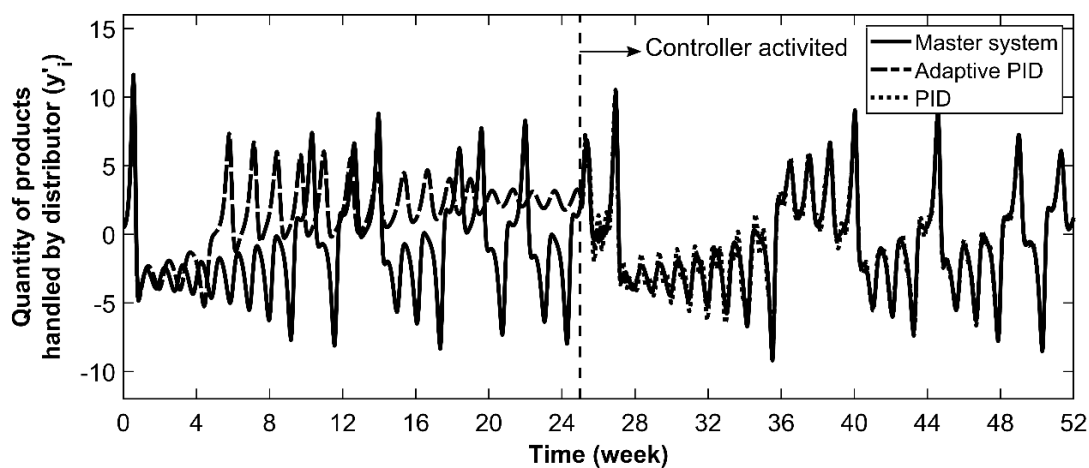
Chaos synchronization policy

The synchronization scheme requires the supply chain system, where information or data should be collected, analyzed, and utilized in real-time to guarantee constant visibility. For short-term periods, sudden changes, such as chaotic surges in demand, can occur unexpectedly in the supply chain networks. Then, the synchronization strategy can help policymakers keep informed about the company's internal stock level with the current picture at the various stages to ensure increased sales and profits with less control action. For this scenario, the controller's initial gains are chosen as follows: $(K_{px}, K_{ix}, K_{dx}) = (1, 0.9, 0.75)$; $(K_{py}, K_{iy}, K_{dy}) = (1.5, 0.35, 1.5)$ and $(K_{pz}, K_{iz}, K_{dz}) = (2, 0.15, 0.5)$. The classical PID controller's initial gains are chosen as follows: $(K_{px_r}, K_{ix_r}, K_{dx_r}) = (1, 0.75, 0.5)$; $(K_{py_r}, K_{iy_r}, K_{dy_r}) = (1.5, 0.2, 1)$ and

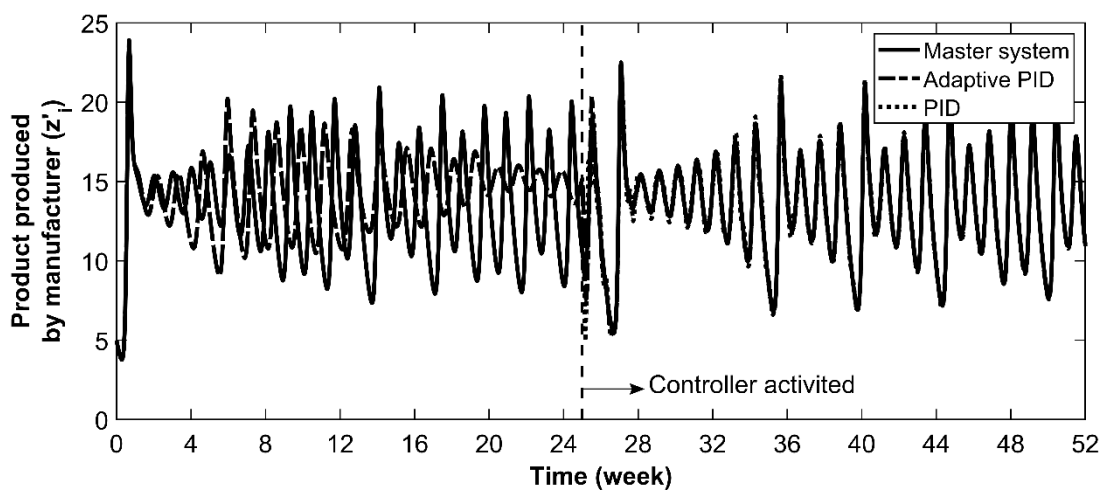
$(K_{pz_r}, K_{iz_r}, K_{dz_r}) = (2, 0.3, 0.75)$. Other parameters of the system have the same values of being used in chaos suppression: $(\gamma, \sigma, \varepsilon, \theta) = (10; 5; 15; -0.2)$. The controller is activated at the 25th week over 52 weeks for all simulations. The synchronization scheme is intended to drive the slave system to follow the ideal system in the disturbance. Fig. 9 shows the errors between the slave system and the master system. As depicted in the simulation results (Figs. 8-10), the proposed adaptive control strategy provides superior performance and accuracy over the classical control strategy. Policymakers need to make decisions quickly and efficiently in a rapidly changing world. From this synchronization approach, making good and timely management decisions can make your business more successful by ensuring competitive advantage by growing revenue and increasing your customer in a volatile market environment.



(a)



(b)



(c)

Fig. 8. Time responses of state variables for supply chain synchronization with control input at the 25th week: (a) product retailer shipping to the customer, (b) quantity of products handled by the distributor, and (c) product produced by the manufacturer.

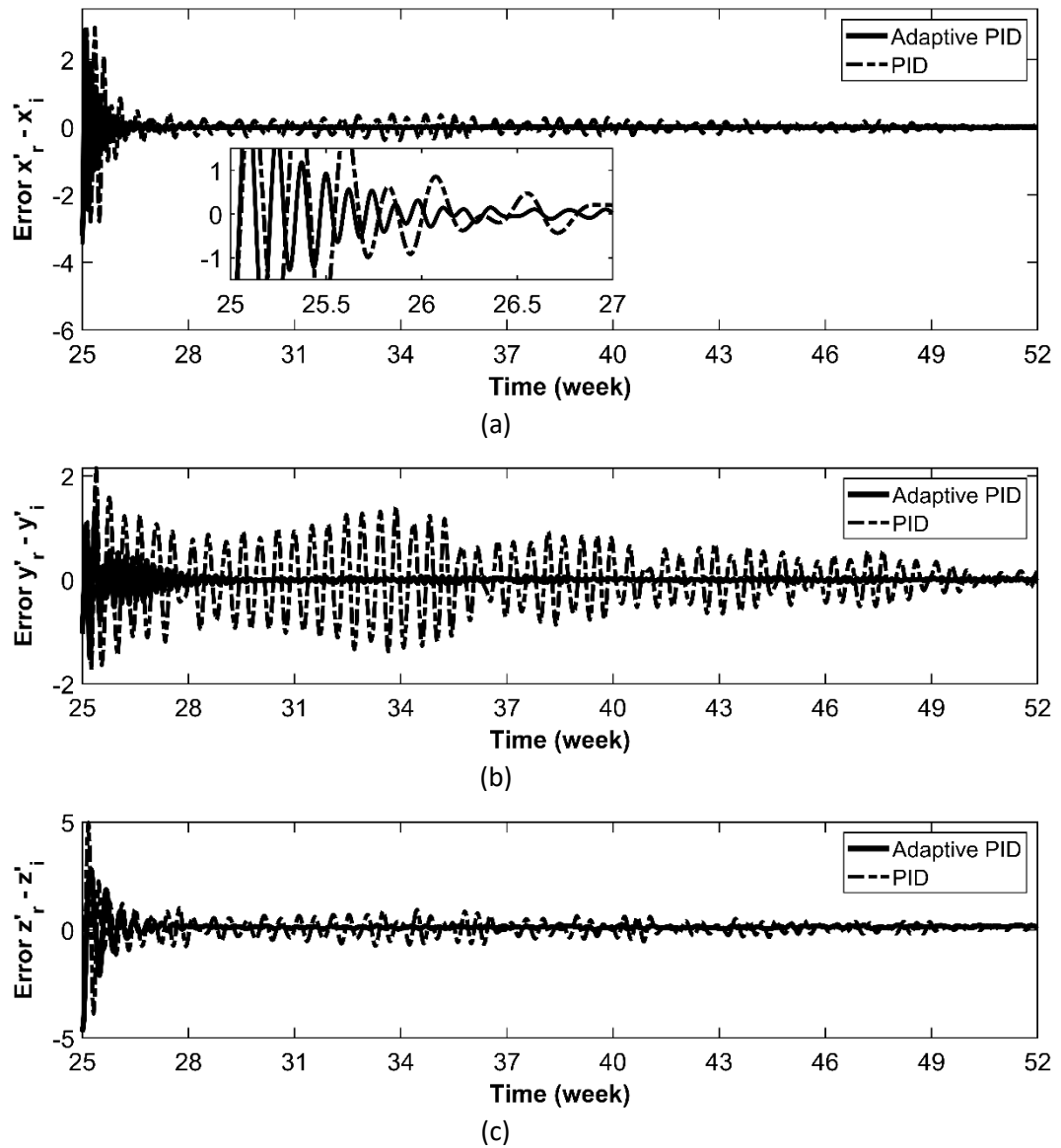
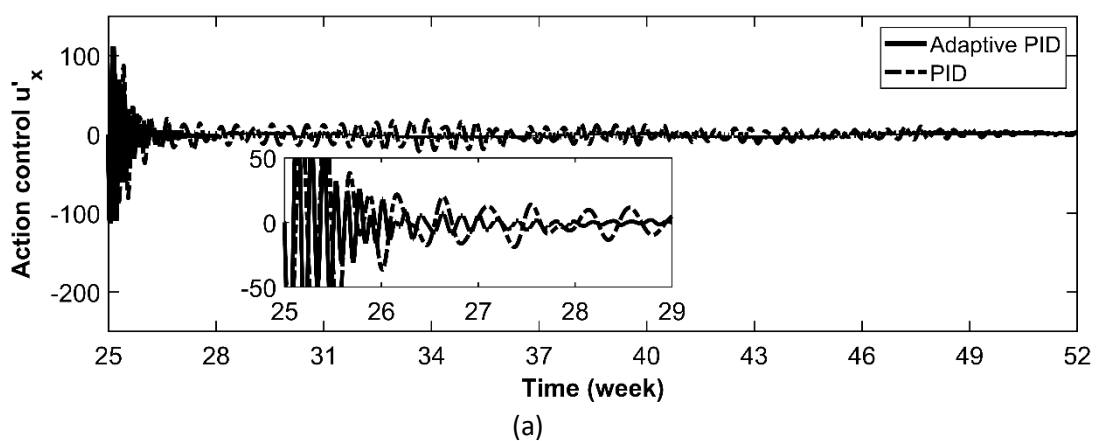


Fig. 9. Time responses of tracking errors for supply chain synchronization with control action at the 25th week: (a) tracking error of the retailer, (b) tracking error of the distributor, and (c) tracking error of the manufacturer.



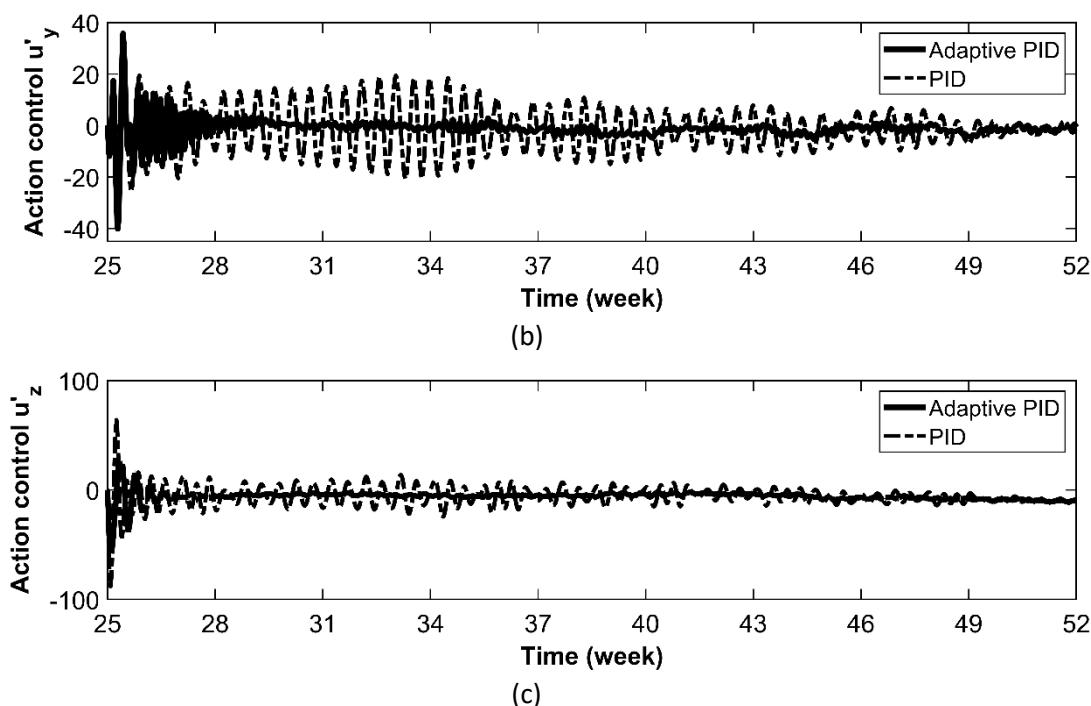


Fig. 10. Control activity (or energy expenditure) of control laws for synchronization policy: (a) control action at the retailer, (b) control action at the distributor, and (c) control action at the manufacturer.

DISCUSSION AND CONCLUSION

This paper proposes a novel adaptive management strategy to regulate uncertain supply chain systems against market disruptions. First, a multiple-echelon system is built with the framework of nonlinear dynamics in a discrete event under stochastic disturbances. The chaotic system describes the complex relationship and dynamic integration between the critical supply chain elements for short-term processes. In reality, the market is always affected by uncertain disrupting factors, which cause stochastic effects on the system. The efficient management strategy helps policymakers implement a decision-support system in a more precise and timely way. A new PID-based adaptive controller has been proposed to realize the chaos suppression and synchronization strategy. The adaptive law is designed based on the gradient descent method so that it can readily update the control gains of the decision-making strategy. Extensive numerical simulations are performed to verify the proposed control strategy. As illustrated in the various test scenarios, the proposed adaptive control strategy provides superior performance and accuracy over the classical control strategy. Cuong et al. (2021) presented adaptive fractional-order

sliding mode control synthesis for solving production-distribution problems in supply chain management. Xu et al. (2020) proposed adaptive sliding mode control for chaos suppression and synchronization in supply chain systems. Amir et al. (2020) proposed a computational method to find optimal PID controller gains. This article presents an adaptive PID algorithm for decision-making policy, in which control gains are updated based on tracking errors in supply chain behaviors. The proposed algorithm is easy to understand and implement, as it employs simple concepts such as tracking errors and adaptive mechanisms to deal with chaotic supply systems against disruptions. Based on the system dynamics and control theory, this study contributes to analyzing, integrating, and controlling the digitized management systems with chaotic behaviors in many of today's supply chains. Finally, the proposed management solutions will help logistics providers prepare for the future and gain a competitive advantage, guaranteeing business profitability against a volatile market.

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Statements and Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

All data generated or analyzed during this study are included in this article (and its supplementary information files).

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